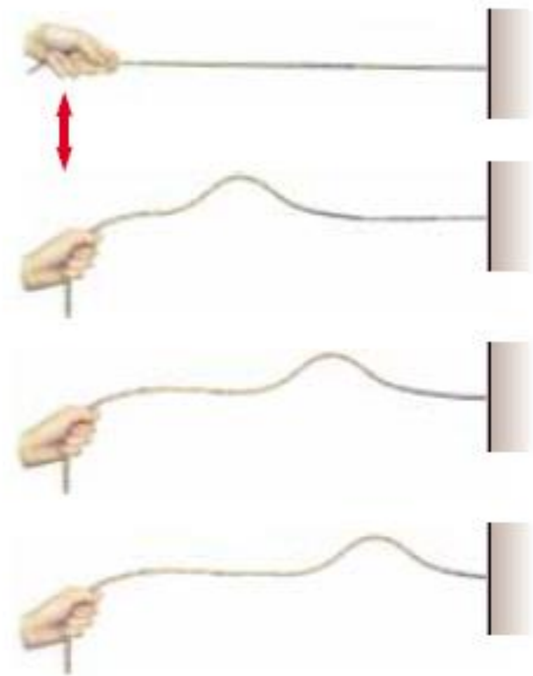
คลื่น (Waves)

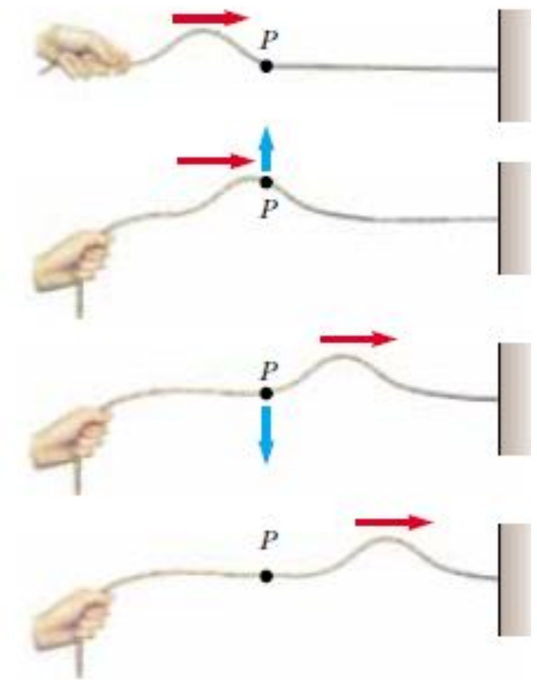
- ชนิดของคลื่น
  - คลื่นกล (Mechanical Waves)
    - อาศัยตัวกลาง เช่น คลื่นน้ำ คลื่นเสียง คลื่นในเส้นเชือก
  - คลื่นแม่เหล็กไฟฟ้า (Electromagnetic waves)
    - ไม่ต้องอาศัยตัวกลาง เช่น คลื่นแสง คลื่นวิทยุ คลื่นไมโครเวฟ

คลื่น (Waves)

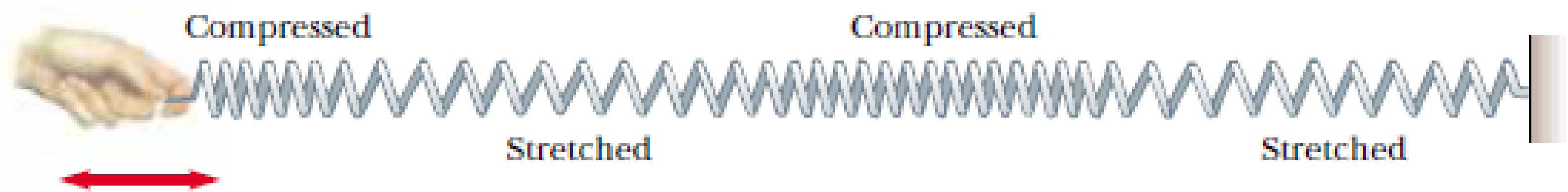
- ชนิดของคลื่น
  - คลื่นตามขวาง (Transverse Waves)
    - คลื่นที่เคลื่อนที่ในทิศตั้งฉากกับทิศการสั่นของตัวกลางหรือตัวสั่น เช่น คลื่นในเส้นเชือก คลื่นน้ำ
  - คลื่นตามยาว (Longitudinal Waves)
    - คลื่นที่เคลื่อนที่ในทิศขนานกับทิศการสั่นของตัวกลางหรือตัวสั่น เช่น คลื่นเสียง

คลื่นตามขวาง

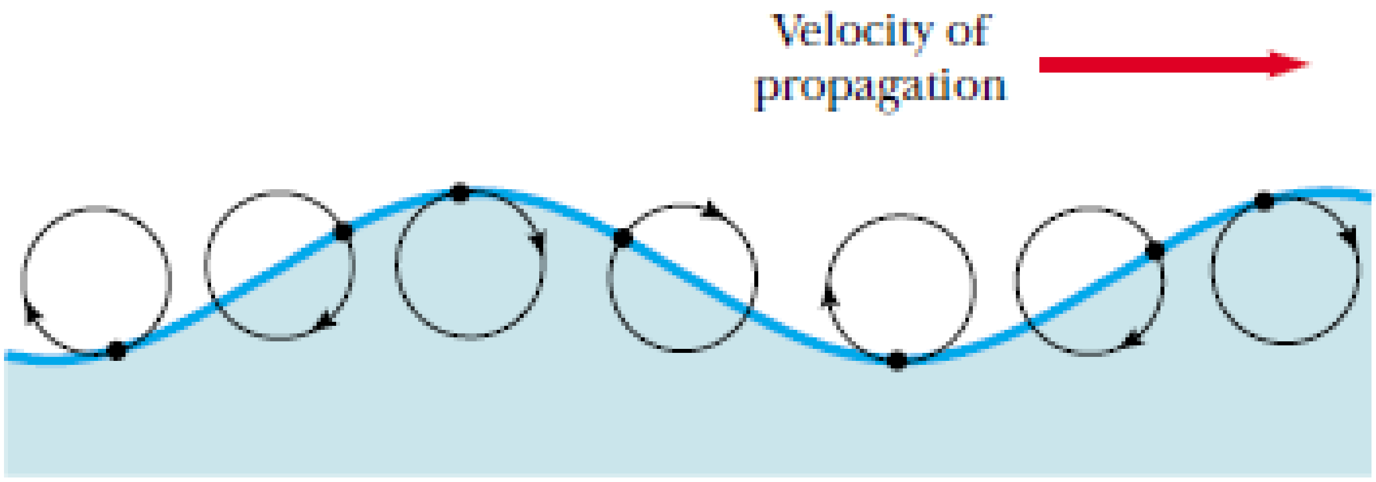




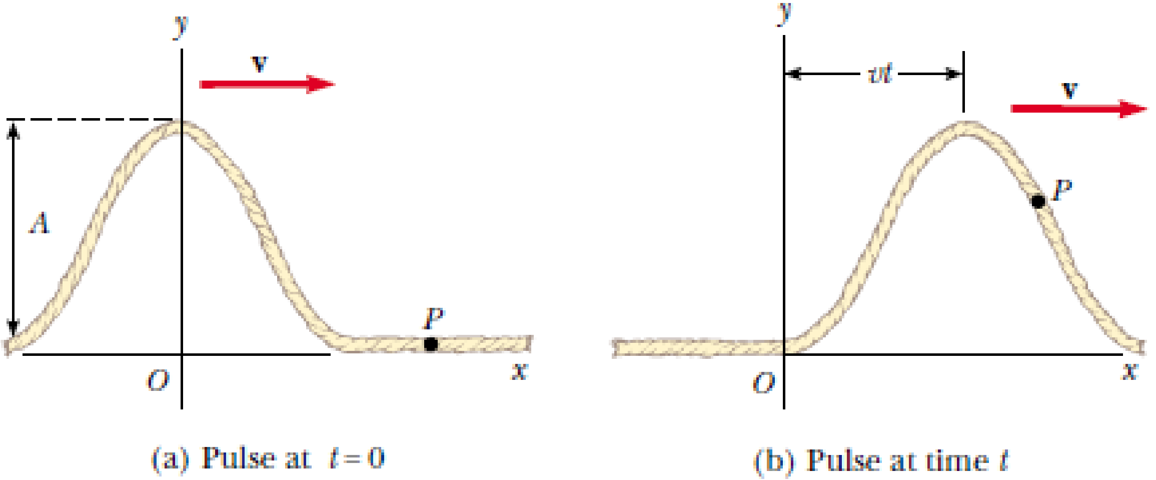
คลื่นตามยาว



การเคลื่อนที่แบบคลื่น



การเคลื่อนที่แบบคลื่น



- ถ้าคลื่นเคลื่อนที่ไปทางขวา

$$y(x,t) = y(x - vt, 0) = f(x - vt)$$

ฟังก์ชันคลื่น (wave function)

การเคลื่อนที่แบบคลื่น

- ถ้าคลื่นเคลื่อนที่ไปทางซ้าย

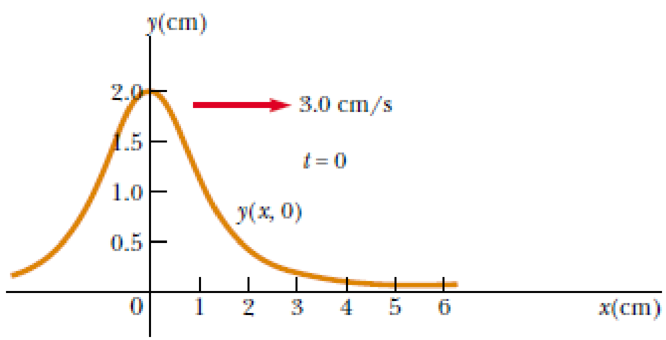
$$y(x,t) = y(x - vt, 0) = f(x + vt)$$

ฟังก์ชันคลื่น (wave function)

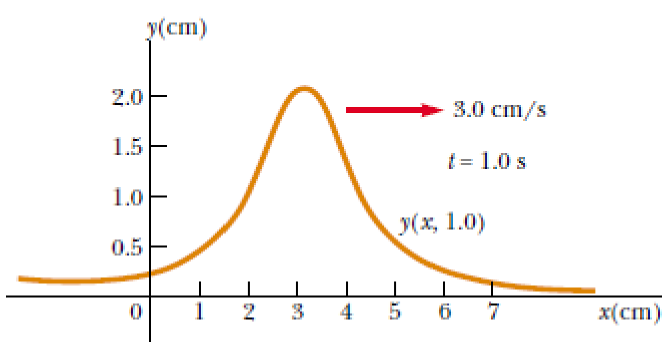
ตัวอย่าง

- คลื่นลูกหนึ่งเคลื่อนที่ไปทางขวาด้วยความเร็ว 3.0 cm/s ดังรูป

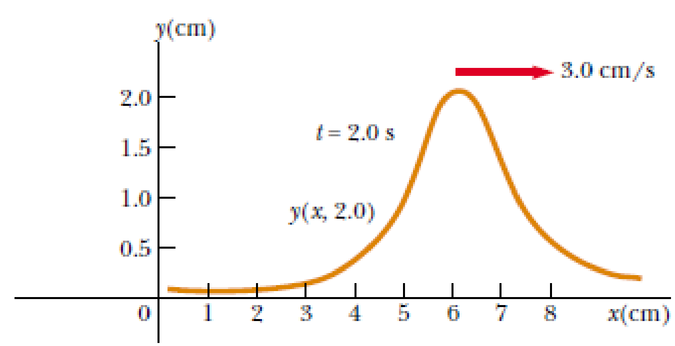
$$y(x,t)=\frac{2}{(x-3t)^2+1}$$



(a)



(b)

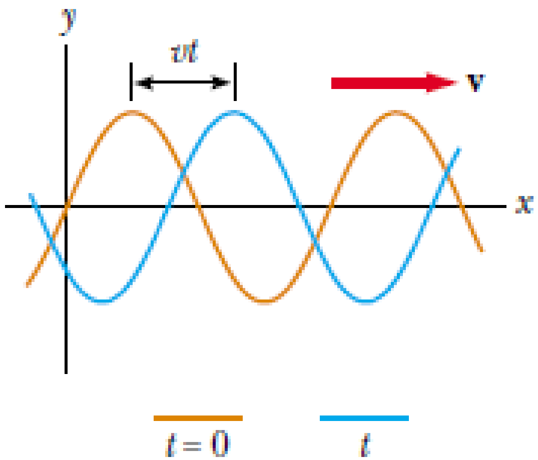


(c)

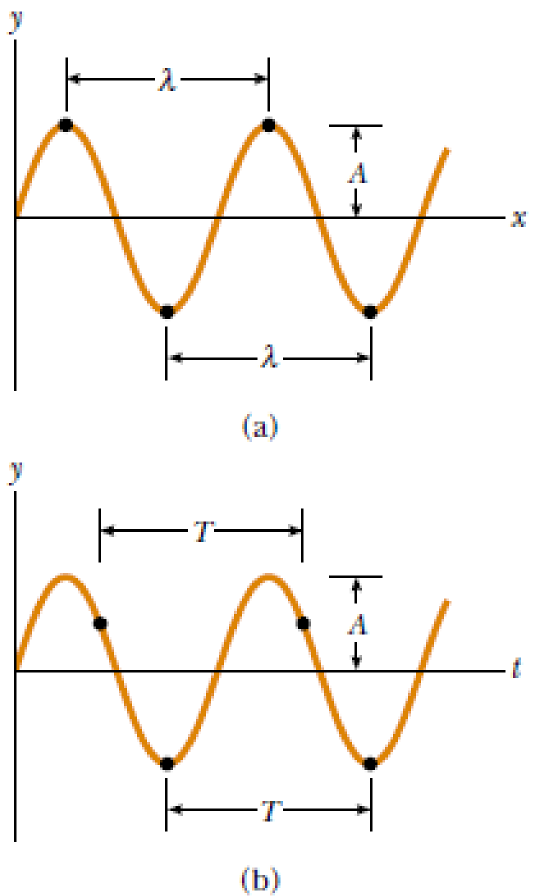
คลื่นแบบไซน์ (Sinusoidal Waves)

- **ความยาวคลื่น** (Wavelength, λ) (m)
  - ระยะทางที่สั้นที่สุดระหว่างจุดสองจุดบนคลื่นที่มีเฟสเดียวกัน
- **คาบ** (Period, T) (s)
  - ช่วงเวลาในการสั่นของตัวกลางครบ 1 รอบ หรือช่วงเวลาที่คลื่นเคลื่อนที่ได้ครบความยาวคลื่น
- **ความถี่** (Frequency, f) (Hz)
  - จำนวนลูกคลื่นที่เคลื่อนที่ผ่านจุดใดจุดหนึ่งในช่วงเวลา 1 วินาที
- **แอมพลิจูด** (Amplitude, A) (m)
  - การขจัดมากสุดของตัวกลางหรือตัวสั่นจากสมดุล

คลื่นแบบไซน์



$$y(x,t) = A\sin\left[\frac{2\pi}{\lambda}(x - vt)\right]$$



คลื่นแบบไซน์

$$y(x,t) = A\sin\left[2\pi\left(\frac{x}{\lambda} - \frac{t}{T}\right)\right]$$

$$f = \frac{1}{T}$$

$$v = \frac{\lambda}{T}$$ ความเร็วคลื่น $$v = \frac{\lambda}{T} = f\lambda$$

$$y(x,t) = A\sin(kx - \omega t)$$

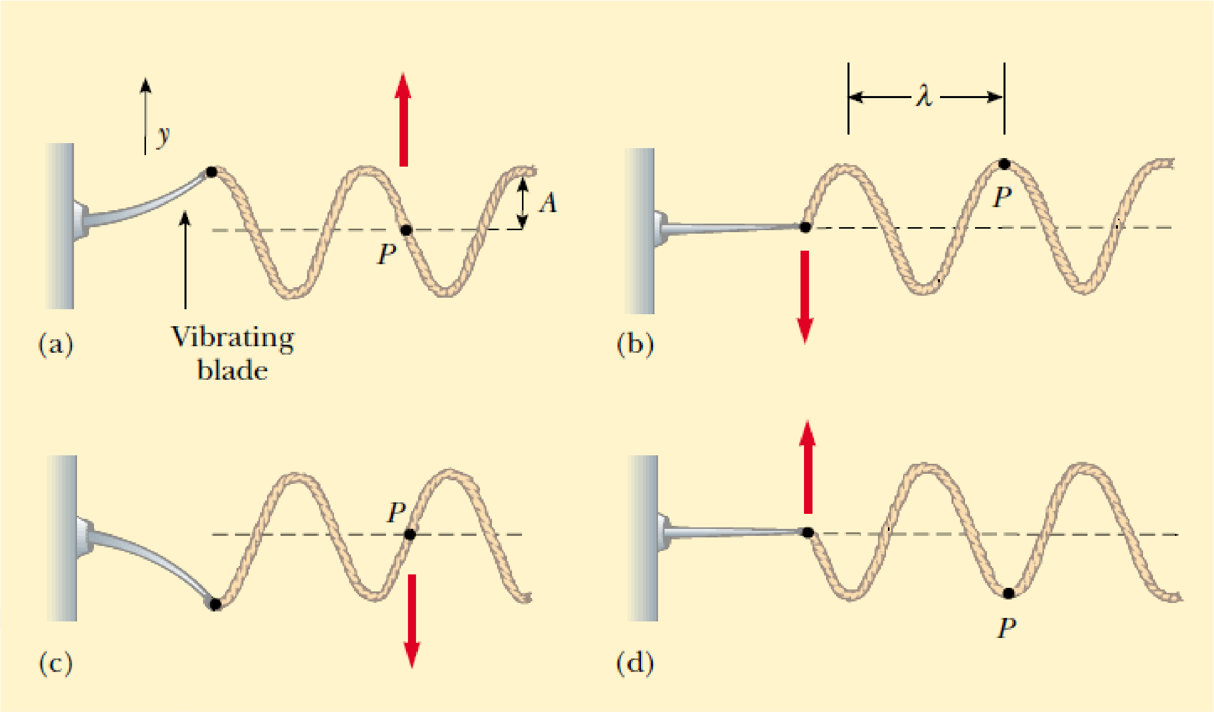
$$k = \frac{2\pi}{\lambda}$$ เลขคลื่น

$$y(x,t) = A\sin(kx - \omega t + \phi)$$

$$\omega = \frac{2\pi}{T}$$ ความถี่เชิงมุม

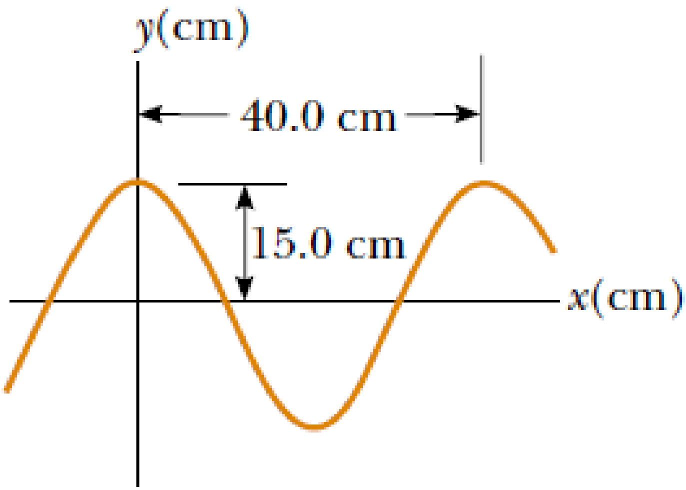
ϕ ค่าคงที่เฟส

คลื่นแบบไซน์ในเส้นเชือก



$$y(x,t) = A\sin\left(kx - \omega t\right)$$

ตัวอย่าง

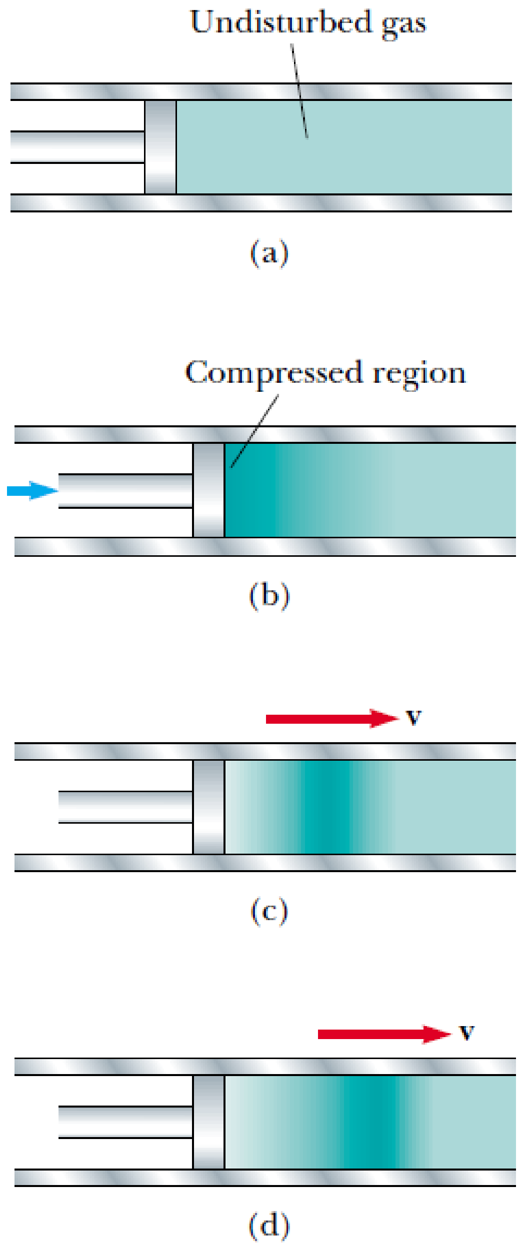


- คลื่นแบบไซน์ขบวนหนึ่ง เคลื่อนที่ไปทางขวา ในแนวแกน X ดังรูป ด้วยอัมปลิจูด 15.0 cm มีความยาวคลื่น 40.0 cm และความถี่เท่ากับ 8.00 Hz ที่เวลา $t = 0$ และ $x = 0$ ลักษณะแสดงดังรูป จงหา
  - เลขคลื่น คาบ ความถี่ และความเร็วคลื่น
  - ค่าคงที่เฟส และสมการของฟังก์ชั่นคลื่น

Sound Waves

- Sound waves are divided into three categories that cover different frequency ranges
  - *Audible waves*
    - lie within the range of sensitivity of the human ear
  - *Infrasonic waves*
    - frequencies below the audible range
  - *Ultrasonic waves*
    - frequencies above the audible range

Speed of Sound Waves



Speed of Sound in Various Media	
Medium	v **(m/s)**
Gases	
Hydrogen (0°C)	1 286
Helium (0°C)	972
Air (20°C)	343
Air (0°C)	331
Oxygen (0°C)	317
Liquids at 25°C	
Glycerol	1 904
Seawater	1 533
Water	1 493
Mercury	1 450
Kerosene	1 324
Methyl alcohol	1 143
Carbon tetrachloride	926
Solids[a]	
Pyrex glass	5 640
Iron	5 950
Aluminum	6 420
Brass	4 700
Copper	5 010
Gold	3 240
Lucite	2 680
Lead	1 960
Rubber	1 600

Speed of Sound Waves

The speed of sound waves in a medium depends on the compressibility and density of the medium

B: bulk modulus
ρ : density of the medium

$$v = \sqrt{\frac{B}{\rho}}$$

$$v = \sqrt{\frac{\text{elastic property}}{\text{inertial property}}}$$

The speed of sound also depends on the temperature of the medium

$$v = (331 \text{ m/s})\sqrt{1 + \frac{T_C}{273°C}}$$

331 m/s is the speed of sound in air at 0°C
T_C is the air temperature in degrees Celsius

Example

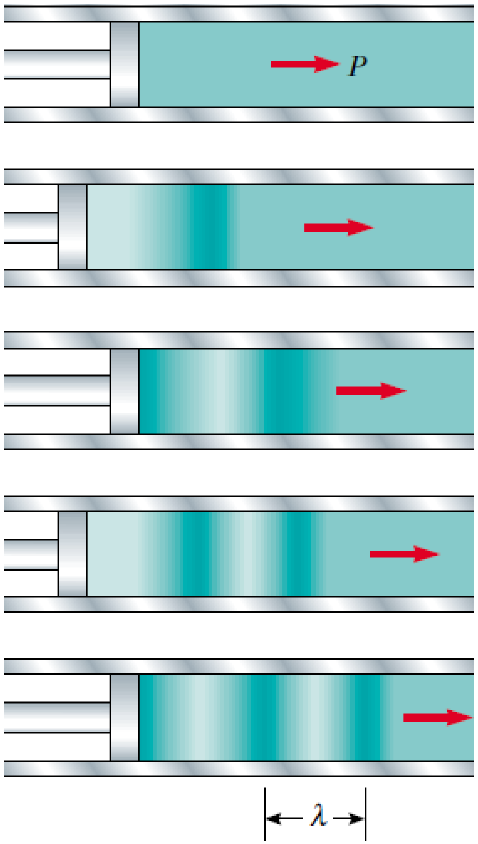
- Find the speed of sound in water, which has a bulk modulus of 2.1×10^9 N/m^2 at a temperature of $0\,^\circ$C and a density of $1.00 \text{ x } 10^3$ kg/m^3.

$$v_{\text{water}} = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{2.1 \times 10^9 \text{ N/m}^2}{1.00 \times 10^3 \text{ kg/m}^3}} = 1.4 \text{ km/s}$$

Periodic Sound Waves

- Compression
  - High-pressure or compressed region
    - The pressure and density in this region fall above their equilibrium
- Rarefactions
  - Low-pressure regions
    - The pressure and density in this region fall below their equilibrium
- Both regions move with a speed equal to the speed of sound in the medium

Periodic Sound Waves

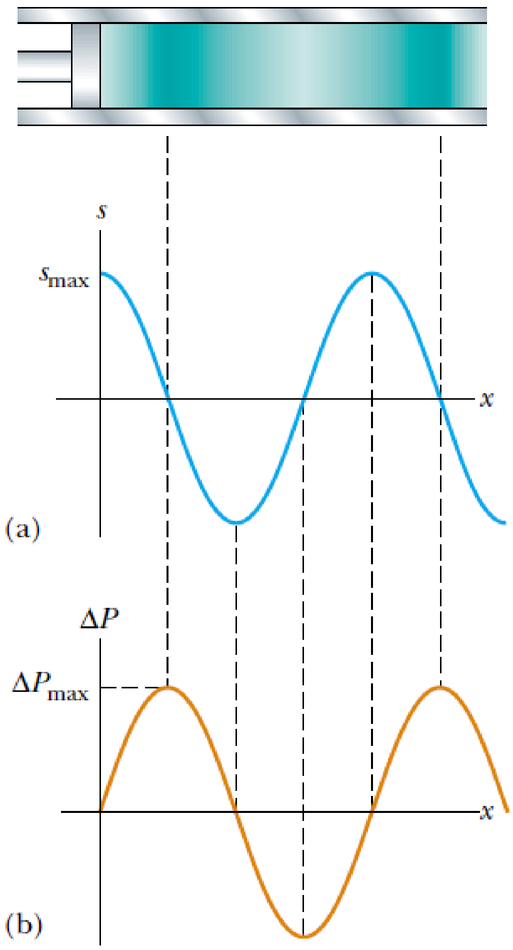


$$s(x, t) = s_{max} \cos(kx - \omega t)$$

$s(x,t)$: the position of a small element relative to its equilibrium position
or harmonic position function

s_{max}: the maximum position of the element relative to equilibrium
or displacement amplitude

Periodic Sound Waves



$$\Delta P = \Delta P_{max} \sin(kx - \omega t)$$

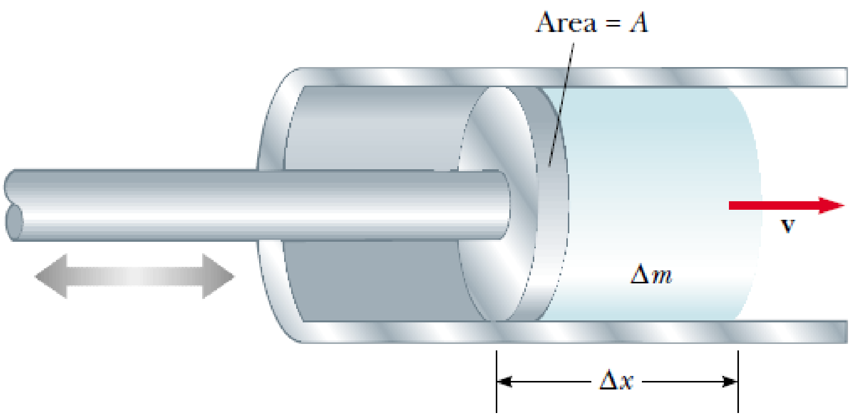
ΔP: the gas pressure measured from the equilibrium value

ΔP_{max}: pressure amplitude

$$\Delta P_{max} = \rho v \omega s_{max}$$

the pressure wave is $90°$ out of phase with the displacement wav

Intensity of Periodic Sound Waves



$$v(x, t) = \frac{\partial}{\partial t} s(x, t) = \frac{\partial}{\partial t} [s_{\max} \cos(kx - \omega t)] = -\omega s_{\max} \sin(kx - \omega t)$$

$$\Delta K = \tfrac{1}{2}\Delta m(v)^2 = \tfrac{1}{2}\Delta m(-\omega s_{\max} \sin kx)^2 = \tfrac{1}{2}\rho A\, \Delta x(-\omega s_{\max} \sin kx)^2$$

$$= \tfrac{1}{2}\rho A\, \Delta x(\omega s_{\max})^2 \sin^2 kx$$

Intensity of Periodic Sound Waves

$$K_\lambda = \int dK = \int_0^\lambda \tfrac{1}{2}\rho A(\omega s_{\max})^2 \sin^2 kx\, dx = \tfrac{1}{2}\rho A(\omega s_{\max})^2 \int_0^\lambda \sin^2 kx\, dx$$

$$= \tfrac{1}{2}\rho A(\omega s_{\max})^2(\tfrac{1}{2}\lambda) = \tfrac{1}{4}\rho A(\omega s_{\max})^2 \lambda$$

$$E_\lambda = K_\lambda + U_\lambda = \tfrac{1}{2}\rho A(\omega s_{\max})^2 \lambda$$

$$\mathscr{P} = \frac{\Delta E}{\Delta t} = \frac{E_\lambda}{T} = \frac{\tfrac{1}{2}\rho A(\omega s_{\max})^2 \lambda}{T} = \tfrac{1}{2}\rho A(\omega s_{\max})^2 \left(\frac{\lambda}{T}\right) = \tfrac{1}{2}\rho A v(\omega s_{\max})^2$$

Intensity of Periodic Sound Waves

$$I \equiv \frac{\mathcal{P}}{A}$$

Intensity (I): the power per unit area or
the rate at which the energy being transported by the wave transfers through a unit area A perpendicular to the direction of travel of the wave

$$I = \frac{\mathcal{P}}{A} = \frac{1}{2}\rho v(\omega s_{\max})^2$$

$$I = \frac{\Delta P_{\max}^2}{2\rho v}$$

Intensity of Periodic Sound Waves

the wave intensity at a distance r from the source

$$I = \frac{\mathcal{P}_{av}}{A} = \frac{\mathcal{P}_{av}}{4\pi r^2}$$

Spherical wave

average power $\mathcal{P}_{av}$ emitted by the source

spherical surface of area $4\pi r^2$.

Sound Level in Decibels

Sound Level in Decibels β

$$\beta \equiv 10 \log\left(\frac{I}{I_0}\right)$$

the *reference intensity*,

$$I_0 = 1.00 \times 10^{-12}\ \mathrm{W/m^2}$$

threshold intensity

Sound Levels

Source of Sound	β (dB)
Nearby jet airplane	150
Jackhammer; machine gun	130
Siren; rock concert	120
Subway; power mower	100
Busy traffic	80
Vacuum cleaner	70
Normal conversation	50
Mosquito buzzing	40
Whisper	30
Rustling leaves	10
Threshold of hearing	0

Example

- Two identical machines are positioned the same distance from a worker. The intensity of sound delivered by each machine at the location of the worker is 2×10^{-7} W/m^2.
  - Find the sound level heard by the worker
    - **(A)** when one machine is operating
    - **(B)** when both machines are operating.

Example

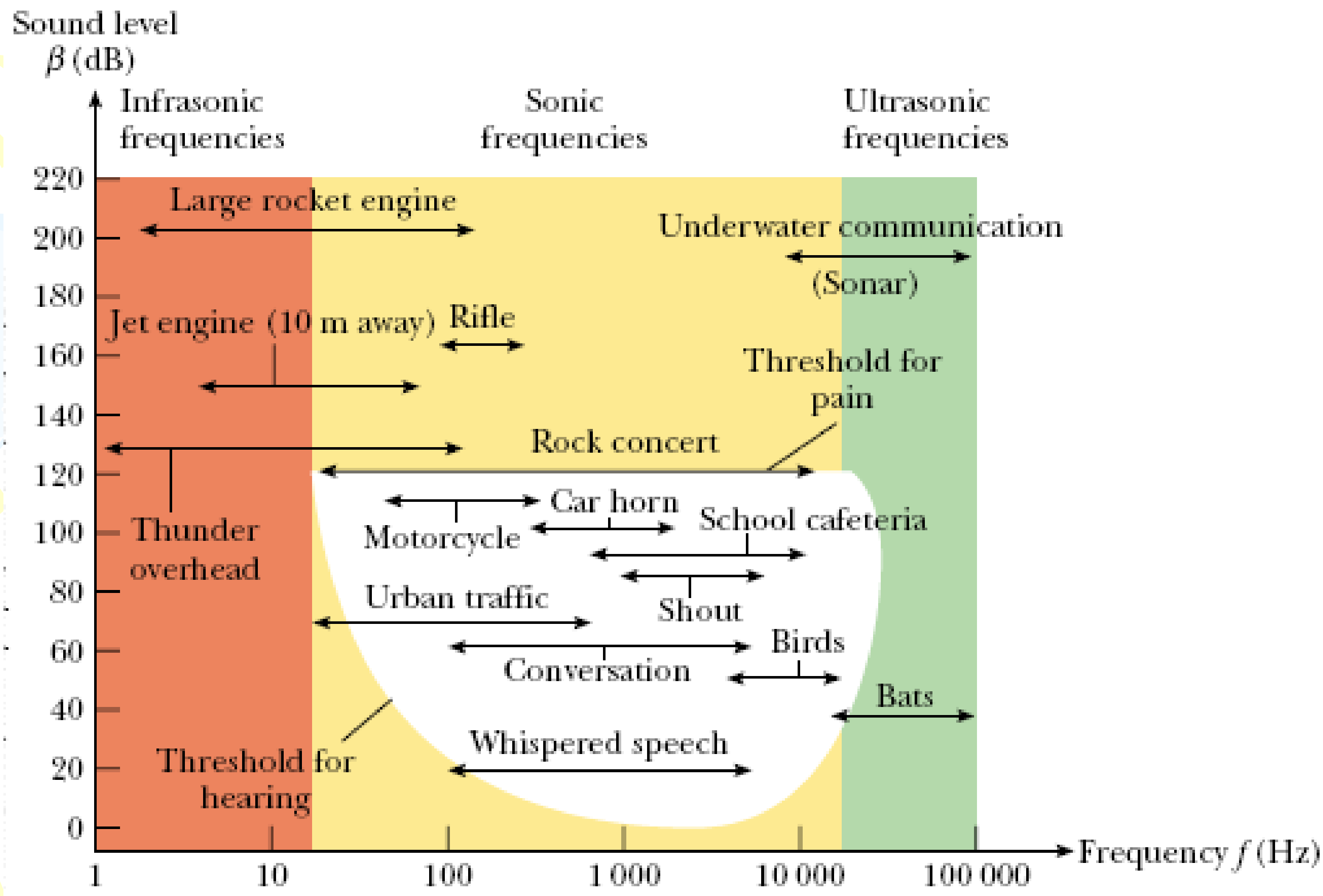
(A) $$\beta_1 = 10\log\left(\frac{2.0\times10^{-7}\,\mathrm{W/m^2}}{1.00\times10^{-12}\,\mathrm{W/m^2}}\right) = 10\log(2.0\times10^{5})$$

$$= \boxed{53\ \mathrm{dB}}$$

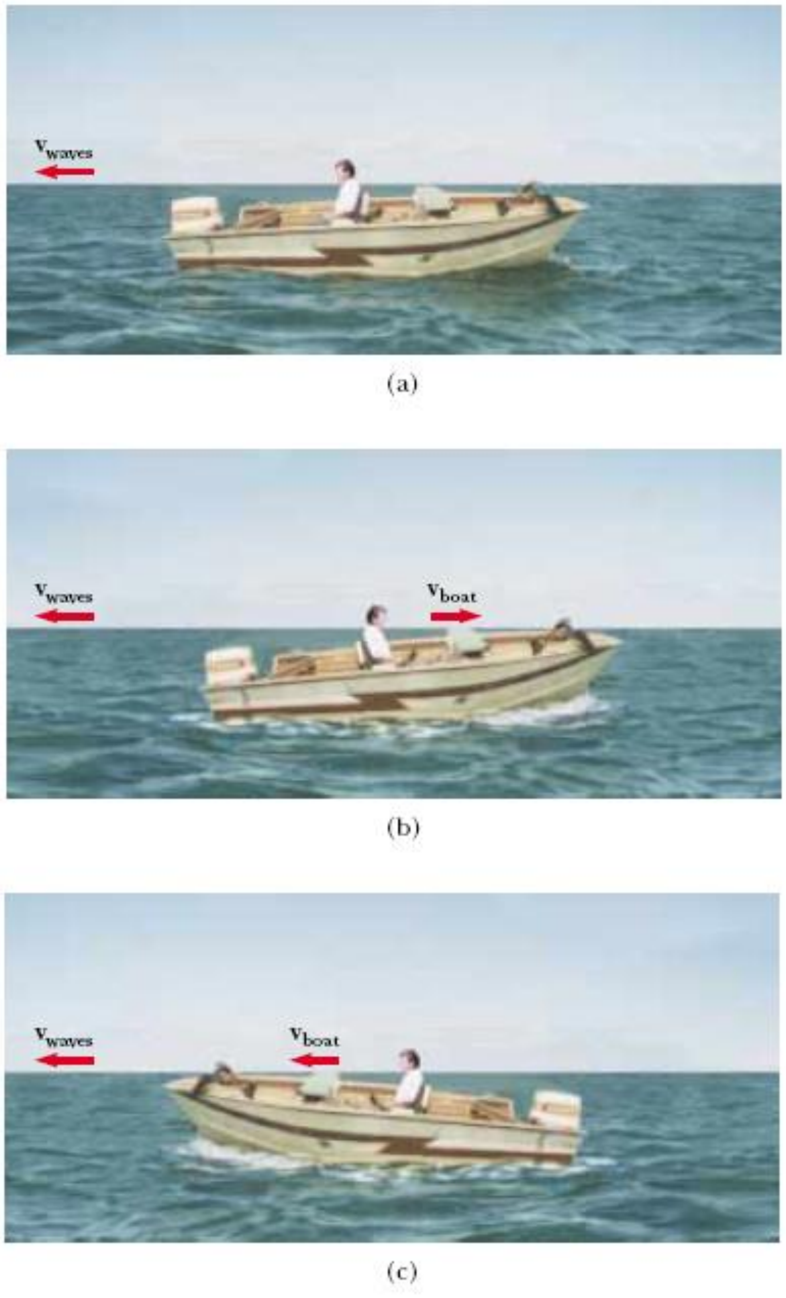
(B) $$\beta_2 = 10\log\left(\frac{4.0\times10^{-7}\,\mathrm{W/m^2}}{1.00\times10^{-12}\,\mathrm{W/m^2}}\right) = 10\log(4.0\times10^{5})$$

$$= \boxed{56\ \mathrm{dB}}$$

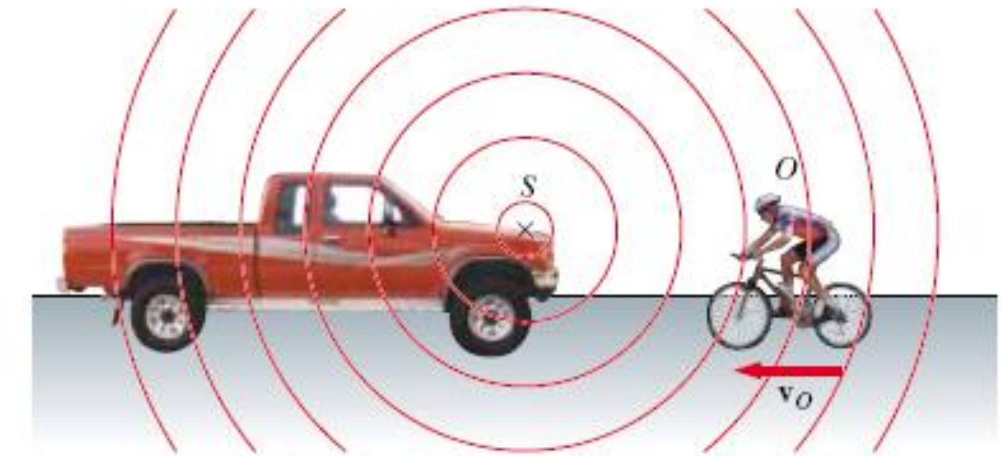
Loudness and Frequency



The Doppler Effect



The Doppler Effect

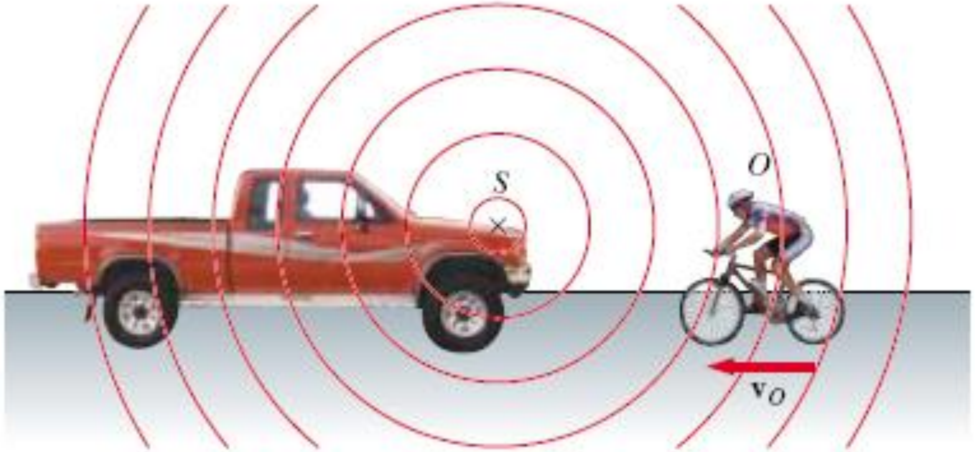


the speed of the waves relative to the observer, *v′*

$$v' = v + v_0$$

speed of sound, *v*
Wavelength, λ with frequency of the source, *f*

The Doppler Effect



the frequency heard by the observer, f′

$$f' = \frac{v'}{\lambda} = \frac{v + v_0}{\lambda}$$

The Doppler Effect

the frequency heard by the observer, f′

$$f' = \frac{v'}{\lambda} = \left(\frac{v \pm v_0}{v}\right) f$$

+ For observer moving toward source
- For observer moving away from source

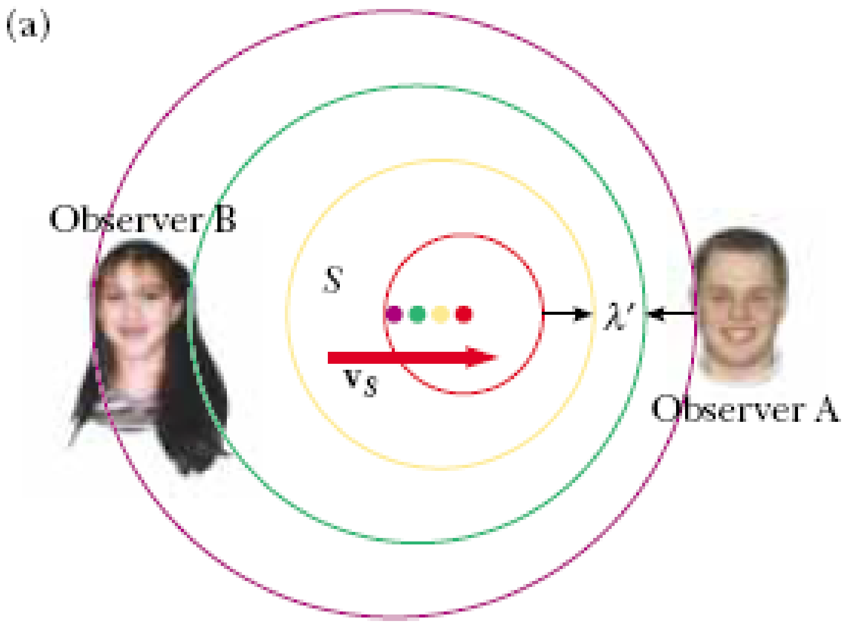
The Doppler Effect

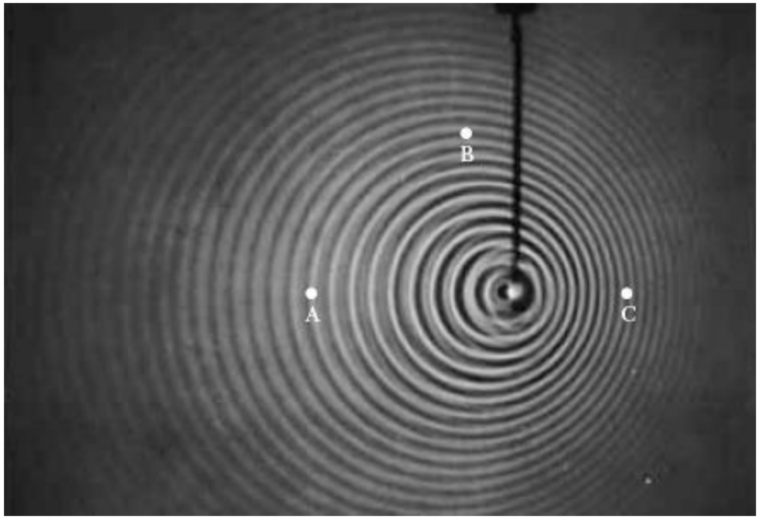
the frequency heard by the observer, f′

$$f' = \frac{v'}{\lambda} = \left(\frac{v \pm v_0}{v} \right) f$$

\+ For observer moving toward source
\- For observer moving away from source

The Doppler Effect





The Doppler Effect

The source in motion and the observer at rest, the frequency heard by the observer, f′

$$f' = \frac{v'}{\lambda} = \left(\frac{v}{v \pm v_s} \right) f$$

+ For source moving away from observer
- For source moving toward observer

Example

- A submarine (sub A) travels through water at a speed of 8.00 m/s, emitting a sonar wave at a frequency of 1,400 Hz. The speed of sound in the water is 1,533 m/s. A second submarine (sub B) is located such that both submarines are traveling directly toward one another. The second submarine is moving at 9.00 m/s.
  - **(A)** What frequency is detected by an observer riding on sub B as the subs approach each other ?
  - **(B)** The subs barely miss each other and pass. What frequency is detected by an observer riding on sub B as the subs recede from each other ?

Example

(A)

$$f' = \left(\frac{v + v_O}{v - v_S}\right) f$$

$$= \left(\frac{1\,533\,\mathrm{m/s} + (+9.00\,\mathrm{m/s})}{1\,533\,\mathrm{m/s} - (+8.00\,\mathrm{m/s})}\right)(1\,400\,\mathrm{Hz}) = \boxed{1\,416\,\mathrm{Hz}}$$

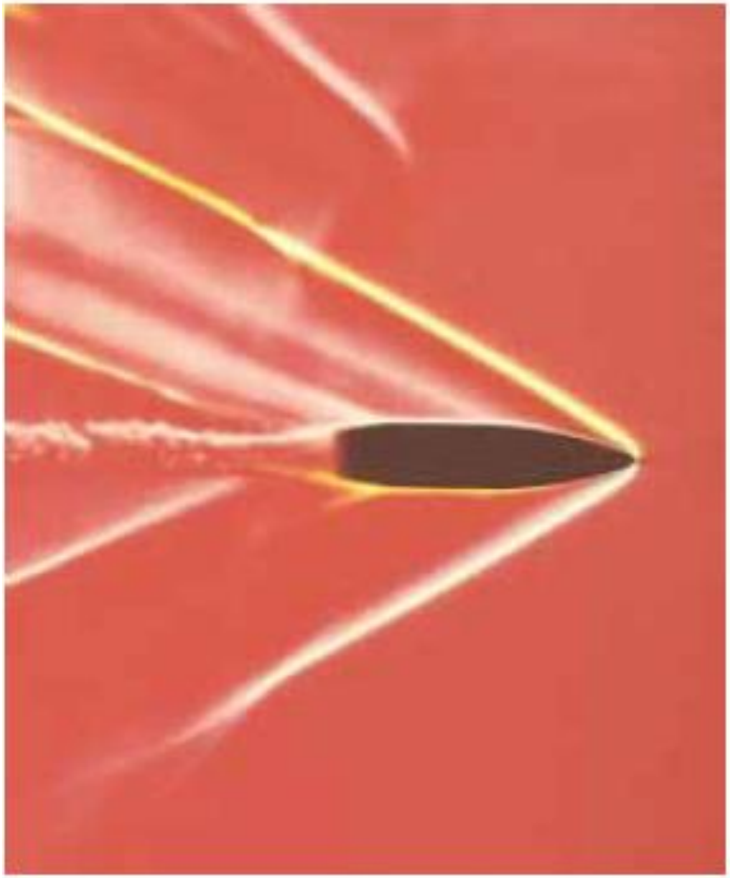
(B)

$$f' = \left(\frac{v + v_O}{v - v_S}\right) f$$

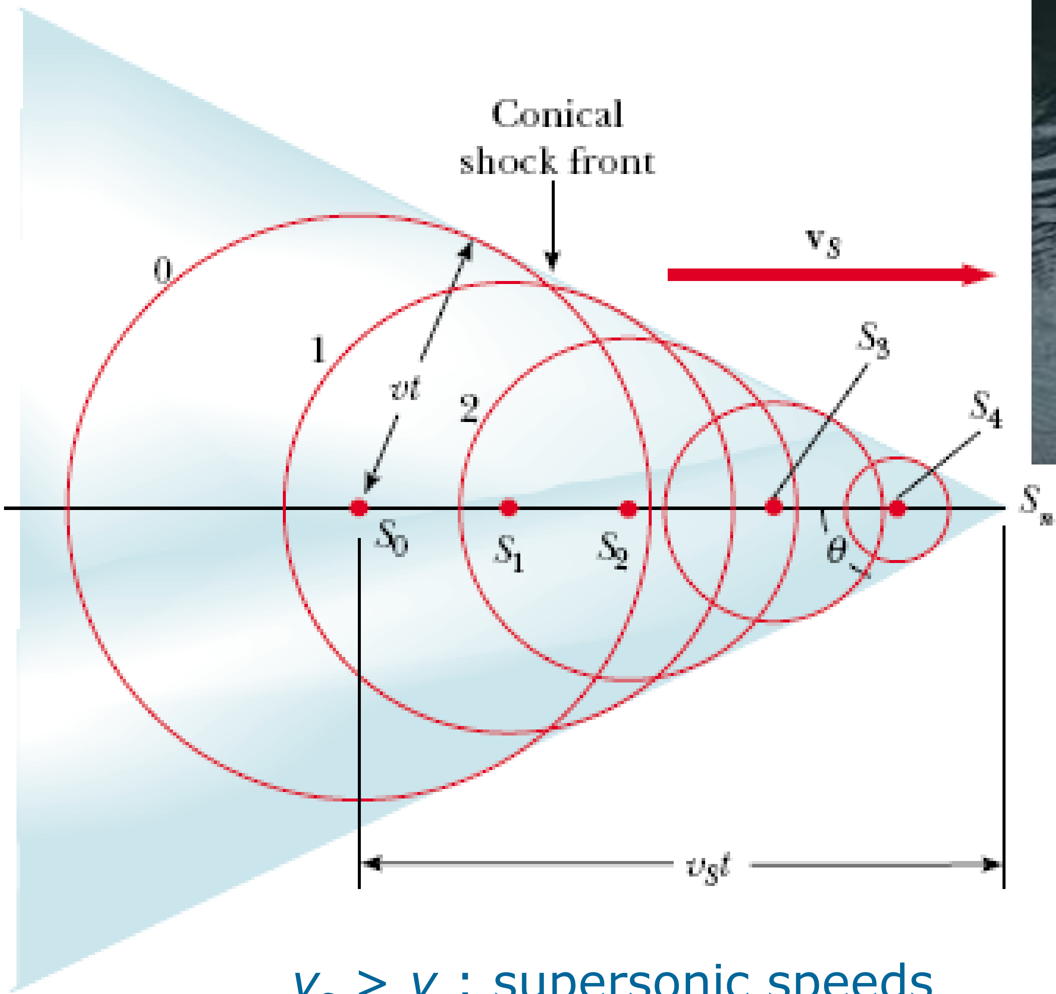
$$= \left(\frac{1\,533\,\mathrm{m/s} + (-9.00\,\mathrm{m/s})}{1\,533\,\mathrm{m/s} - (-8.00\,\mathrm{m/s})}\right)(1\,400\,\mathrm{Hz}) = \boxed{1\,385\,\mathrm{Hz}}$$

Shock Waves

- speed of a source *exceeds* the wave speed
- **V**-shaped wave fronts
- conical wave front



Shock Waves



$v_s > v$: supersonic speeds

$$\sin\theta = \frac{vt}{v_s t} = \frac{v}{v_s}$$

Mach number

Superposition

- Superposition principle
  - If two or more traveling waves are moving through a medium, the resultant value of the wave function at any point is the algebraic sum of the values of the wave functions of the individual waves.
- Two traveling waves can pass through each other without being destroyed or even altered

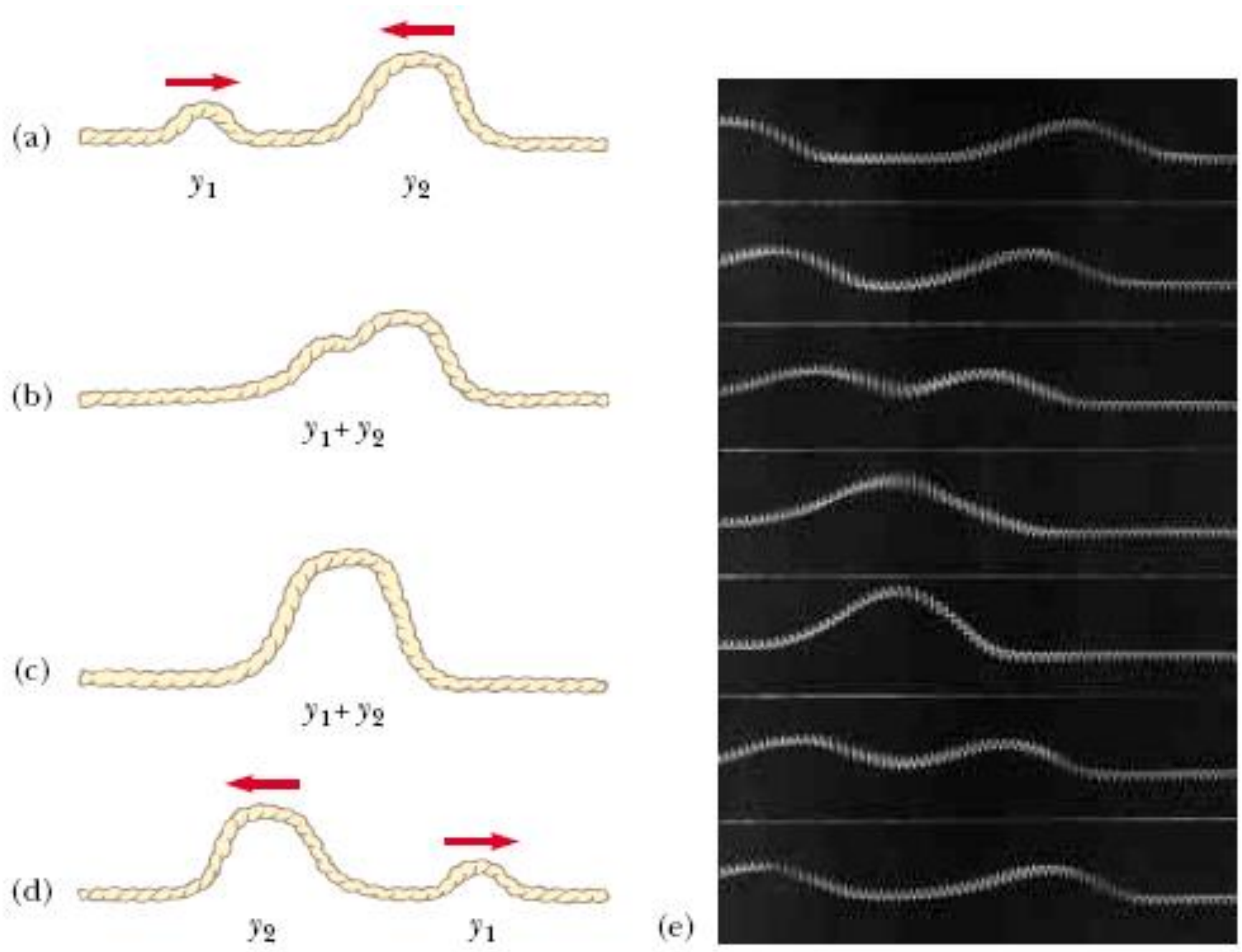
Superposition

- Superposition principle
  - If two or more traveling waves are moving through a medium, the resultant value of the wave function at any point is the algebraic sum of the values of the wave functions of the individual waves.
- Two traveling waves can pass through each other without being destroyed or even altered

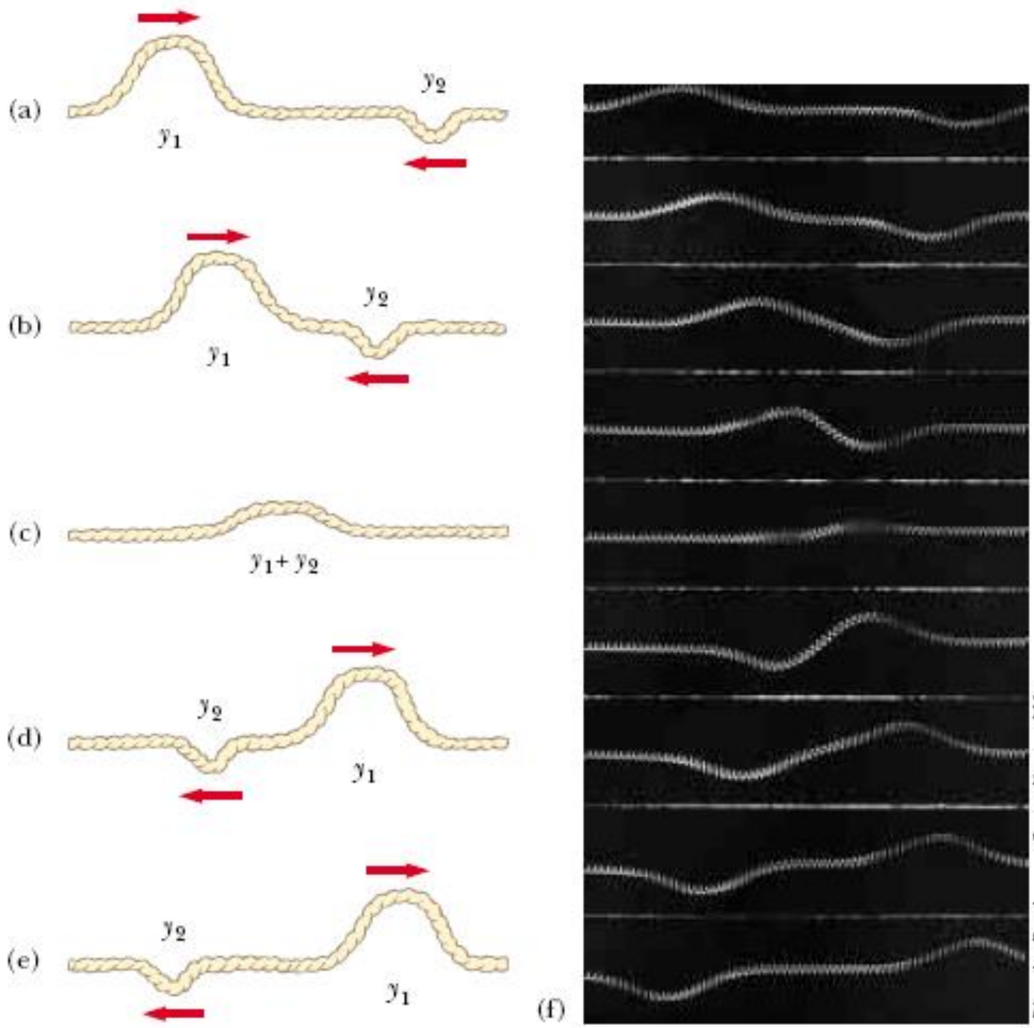
Interference

- Interference
  - the combination of separate waves in the same region of space to produce a resultant wave
- Constructive interference
  - the two pulses in the same direction
- Destructive interference
  - the two pulses in opposite direction

Superposition



Superposition



Superposition of Sinusoidal Waves

$$y_1 = A\sin(kx - \omega t) \qquad y_2 = A\sin(kx - \omega t + \phi)$$

$$y = y_1 + y_2 = A[\sin(kx - \omega t) + \sin(kx - \omega t + \phi)]$$

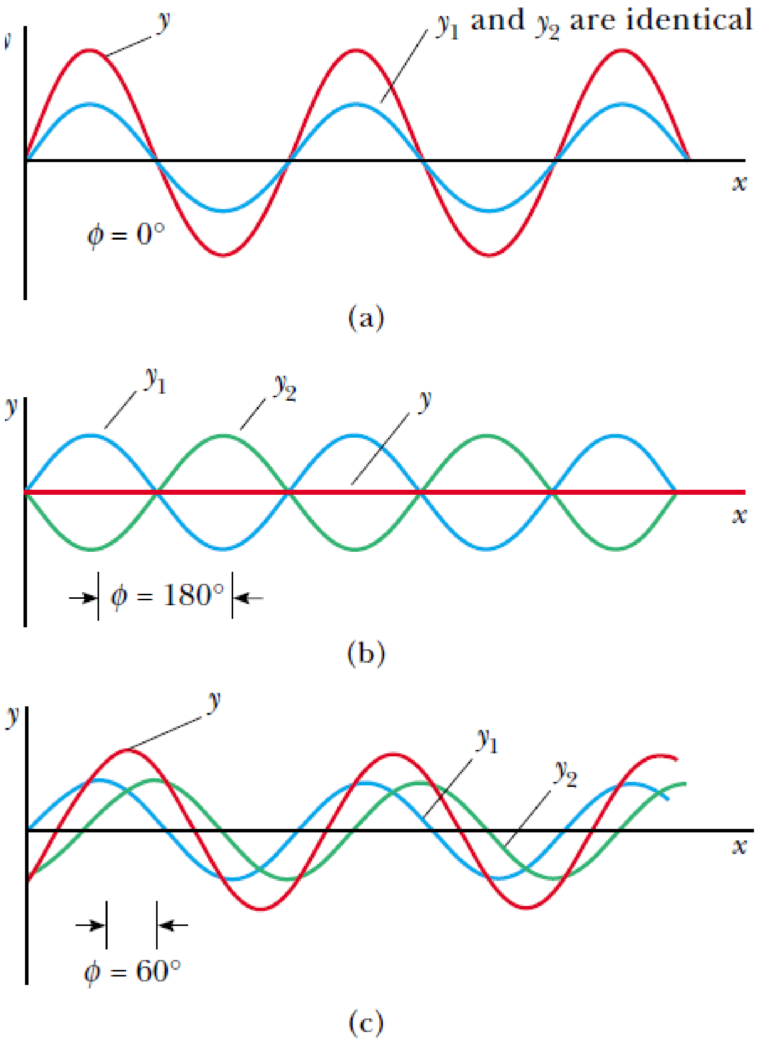
$$\sin a + \sin b = 2\cos\left(\frac{a-b}{2}\right)\sin\left(\frac{a+b}{2}\right)$$

$$a = kx - \omega t$$
$$b = kx - \omega t + \phi$$

⇨

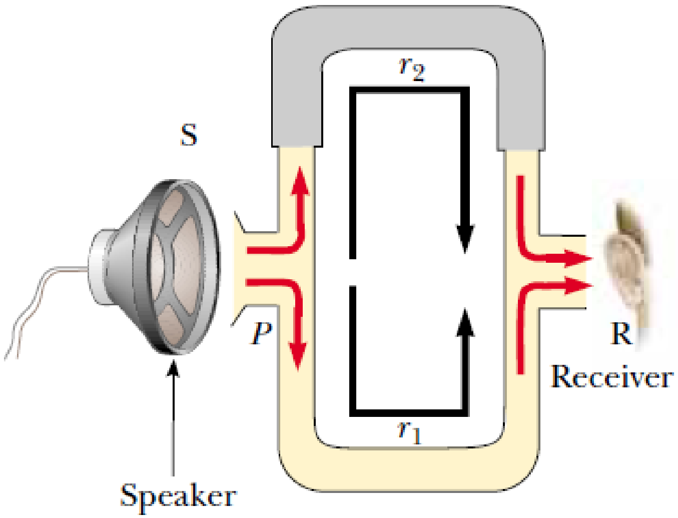
$$y = 2A\cos\left(\frac{\phi}{2}\right)\sin\left(kx - \omega t + \frac{\phi}{2}\right)$$

Superposition of Sinusoidal Waves



- Constructive interference
  - the resultant wave has maximum amplitude
- Destructive interference
  - the resultant wave has *zero* amplitude

Interference of Sound Waves



path length $\Delta r = |r_2 - r_1|$

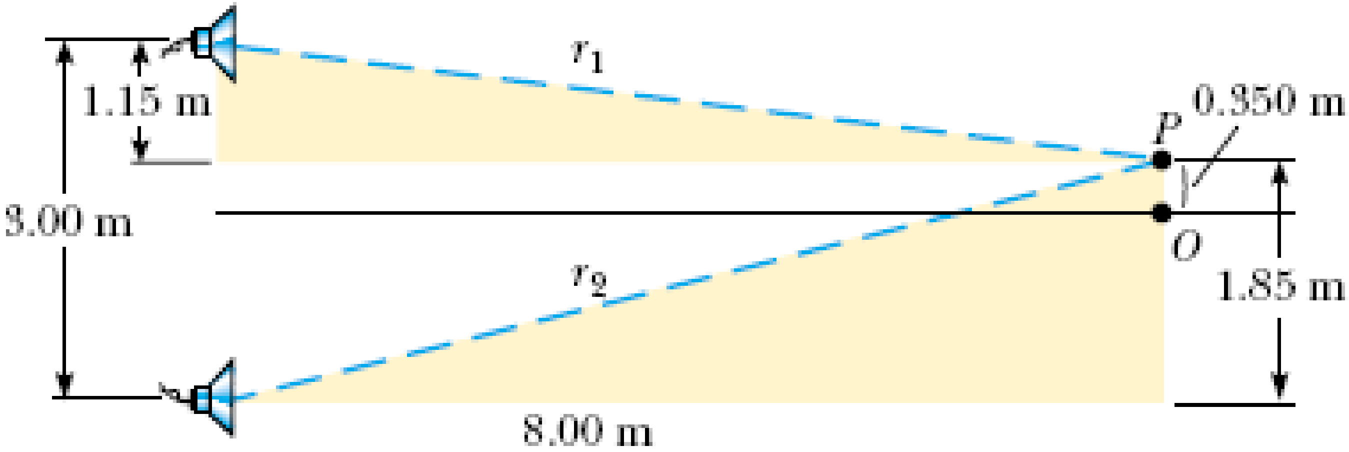
$$\Delta r = \frac{\phi}{2\pi}\lambda$$

$$\Delta r = (2n)\frac{\lambda}{2} \quad \text{for constructive interference}$$

$$\Delta r = (2n+1)\frac{\lambda}{2} \quad \text{for destructive interference}$$

$$n = 0, 1, 2, 3, \ldots,$$

Example



- A pair of speakers placed 3.00 m apart are driven by the same oscillator as shown in the figure. A listener is originally at point *O*, which is located 8.00 m from the center of the line connecting the two speakers. The listener then walks to point *P*, which is a perpendicular distance 0.350m from *O*, before reaching the *first minimum* in sound intensity. What is the frequency of the oscillator ?

Example

$$r_1 = \sqrt{(8.00\text{ m})^2 + (1.15\text{ m})^2} = 8.08\text{ m}$$

$$r_2 = \sqrt{(8.00\text{ m})^2 + (1.85\text{ m})^2} = 8.21\text{ m}$$

$r_2 - r_1 = 0.13\text{ m}$ path difference be equal to $\lambda/2$

$$f = \frac{v}{\lambda} = \frac{343\text{ m/s}}{0.26\text{ m}} = 1.3\text{ kHz}$$

Standing Waves

superposition of two transverse sinusoidal waves having the same amplitude, frequency, and wavelength but traveling in opposite directions in the same medium

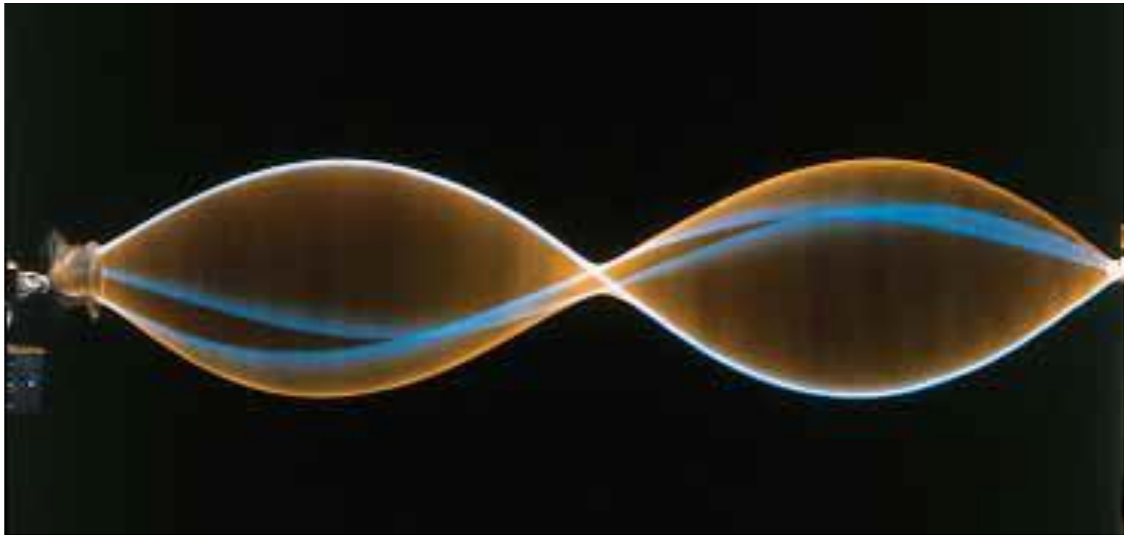
$$y_1 = A\sin(kx - \omega t) \qquad y_2 = A\sin(kx + \omega t)$$

$$y = y_1 + y_2 = A\sin(kx - \omega t) + A\sin(kx + \omega t)$$

$$y = (2A\sin kx)\cos\omega t$$

wave function of a standing wave

Standing Waves



Nodes : points of zero displacement
Antinodes : points of maximum displacement

Standing Waves

Nodes

$$kx = \pi, 2\pi, 3\pi, \ldots$$

$$x = \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, \ldots = \frac{n\lambda}{2} \qquad n = 0, 1, 2, 3, \ldots$$

Standing Waves

Antinodes

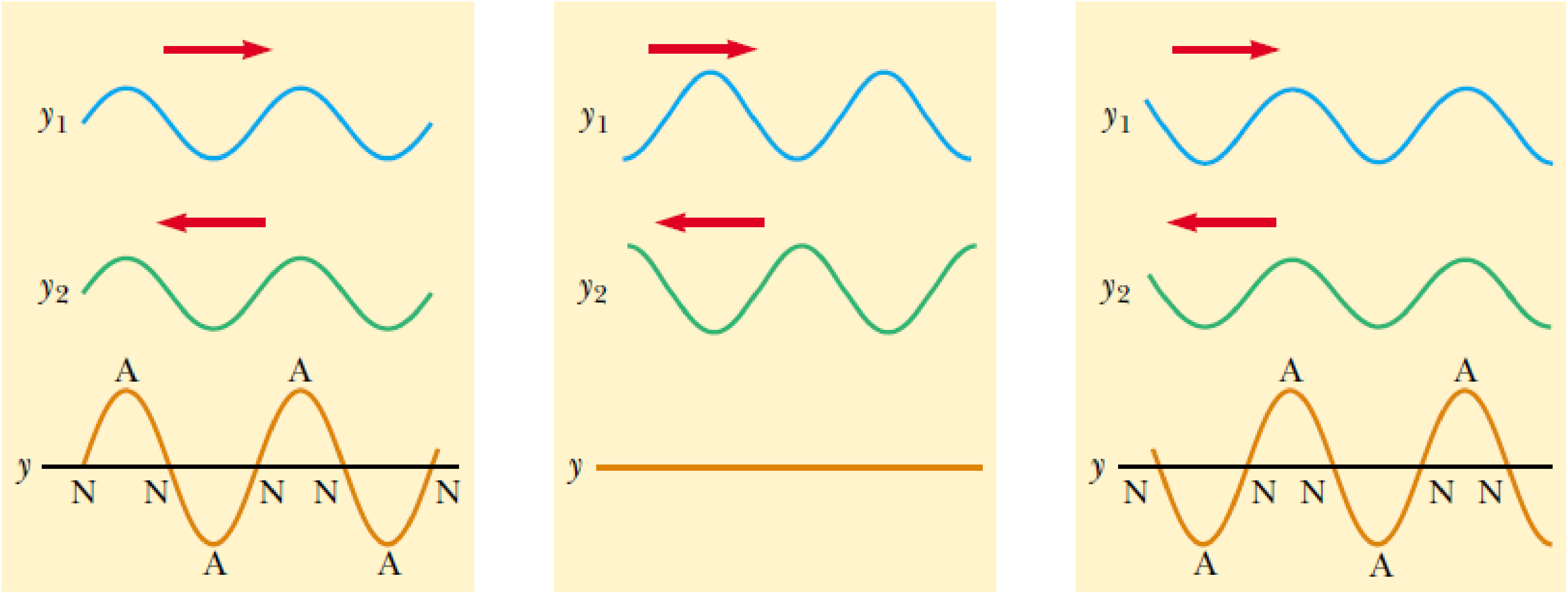
$$kx = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots$$

$$x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \ldots = \frac{n\lambda}{4} \qquad n = 1, 3, 5, \ldots$$

Standing Waves

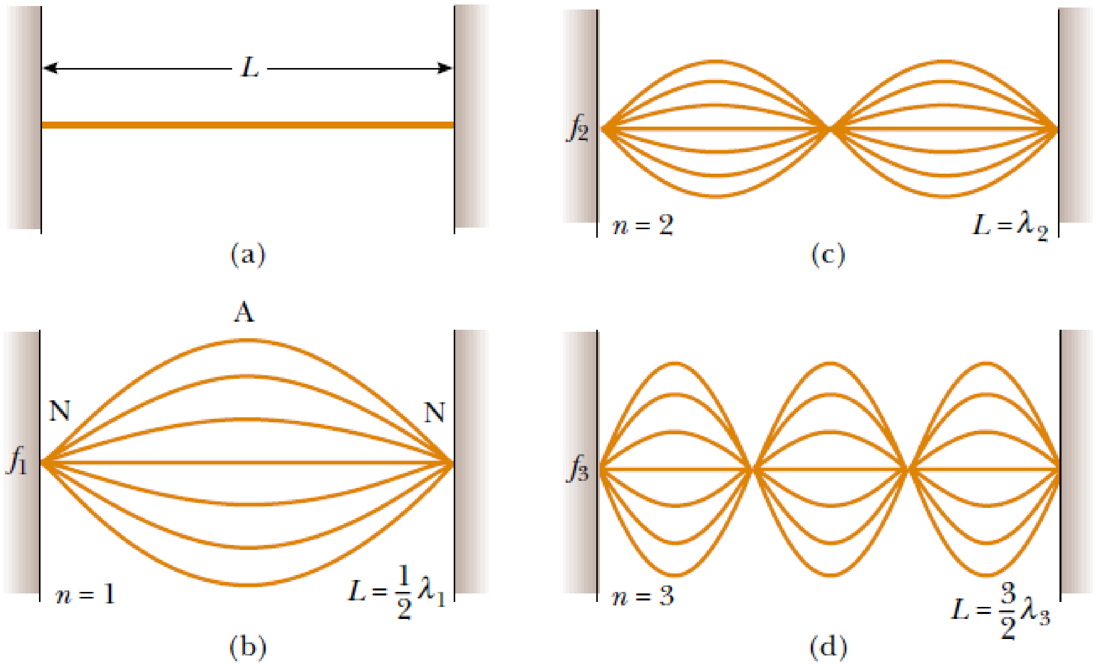
- The distance between adjacent antinodes is equal to $\lambda/2$
- The distance between adjacent nodes is equal to $\lambda/2$
- The distance between a node and an adjacent antinode is $\lambda/4$.

Standing Waves



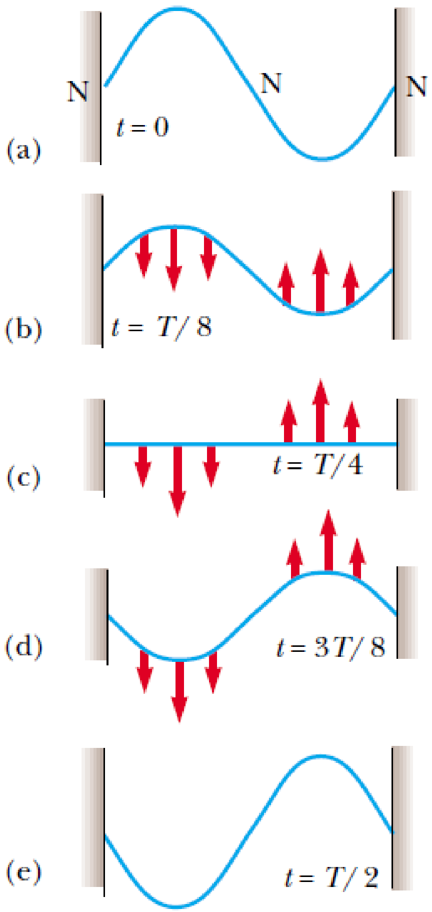
(a) $t = 0$ (b) $t = T/4$ (c) $t = T/2$

Standing Waves in a String Fixed at Both Ends



Normal modes : a number of natural patterns of oscillation

Standing Waves in a String Fixed at Both Ends



wavelengths of the various normal modes

$$\lambda_n = \frac{2L}{n} \qquad n = 1, 2, 3, \ldots$$

Natural frequencies of the normal modes
or, quantized frequencies

$$f_n = \frac{v}{\lambda_n} = n\frac{v}{2L} \qquad n = 1, 2, 3, \ldots$$

Standing Waves in a String Fixed at Both Ends

natural frequencies of a taut string

$$v = \sqrt{T/\mu}$$

$$f_n = \frac{n}{2L}\sqrt{\frac{T}{\mu}} \qquad n = 1, 2, 3, \ldots$$

fundamental frequency

$$f_1 = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

Standing Waves in a String Fixed at Both Ends

- Harmonics
  - Frequencies of normal modes that exhibit an integer-multiple relationship
    - such as this form a harmonic series, and the normal modes

Standing Waves in a String Fixed at Both Ends

boundary conditions

At *x = 0* and *x = L* $y(0,\ t) = 0$ $y(L,\ t) = 0$

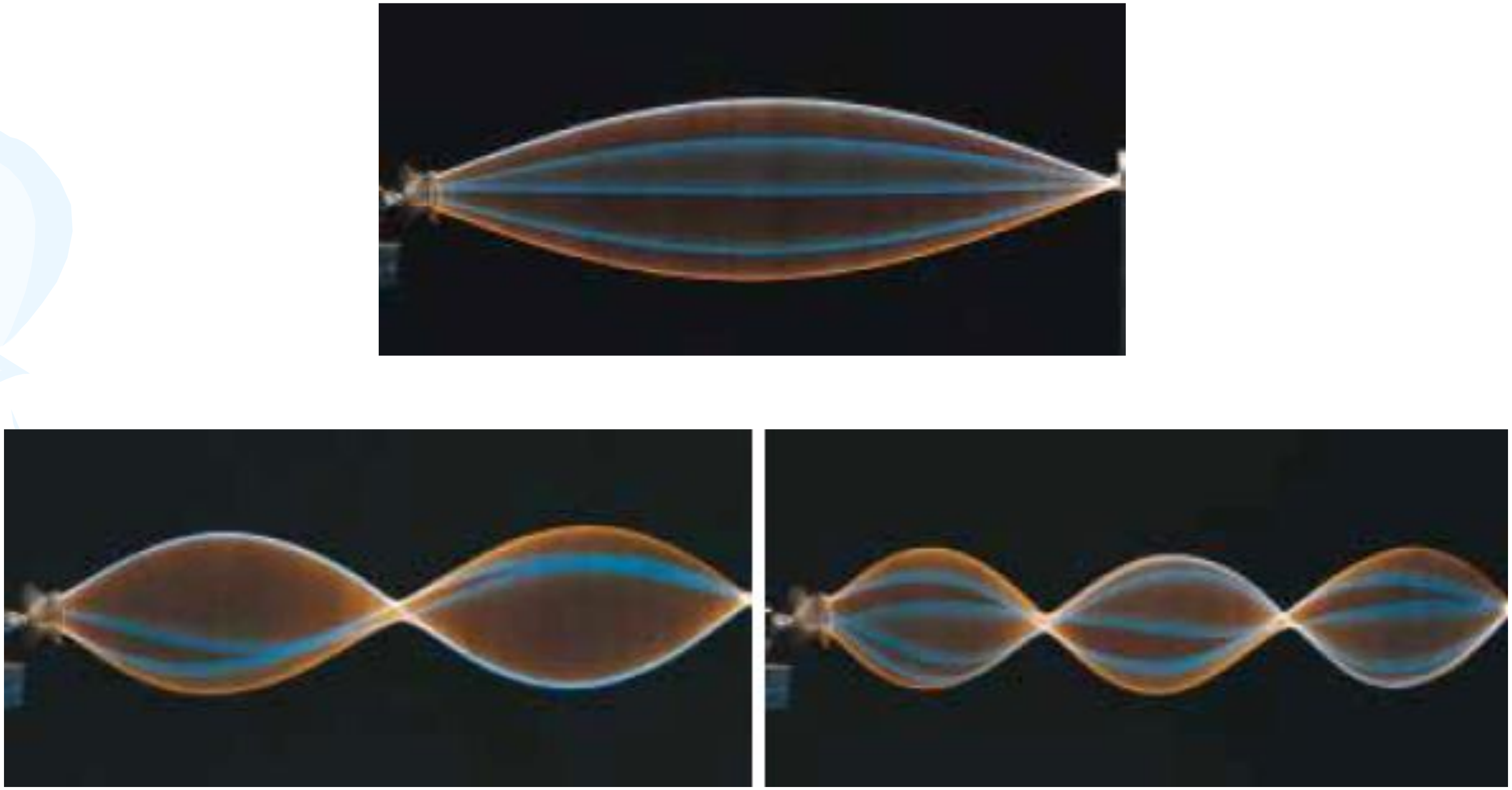
$$y = 2A(\sin kx)\cos \omega t \Longrightarrow \sin kL = 0$$

$$k_n L = n\pi \qquad n = 1, 2, 3, \ldots$$

$$k_n = 2\pi/\lambda_n$$

$$\left(\frac{2\pi}{\lambda_n}\right) L = n\pi \qquad \text{or} \qquad \lambda_n = \frac{2L}{n}$$

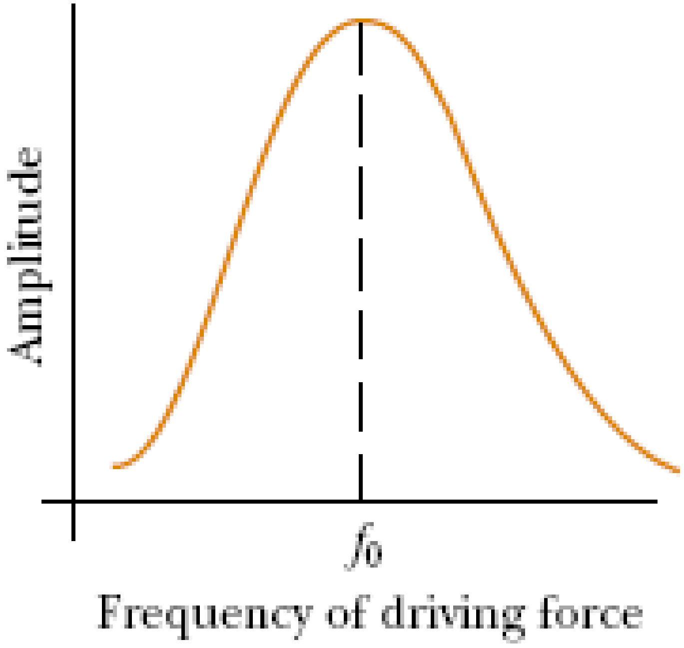
Standing Waves in a String Fixed at Both Ends

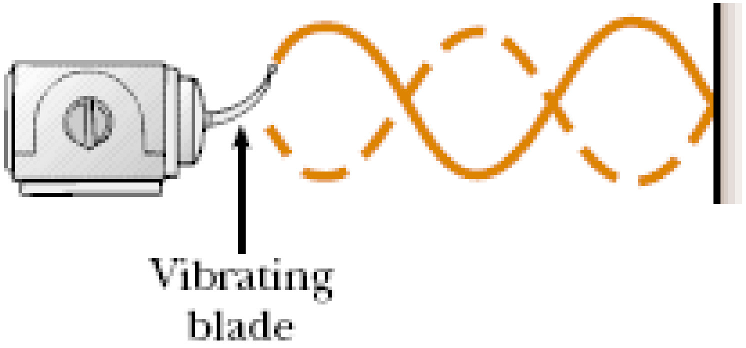


Resonance

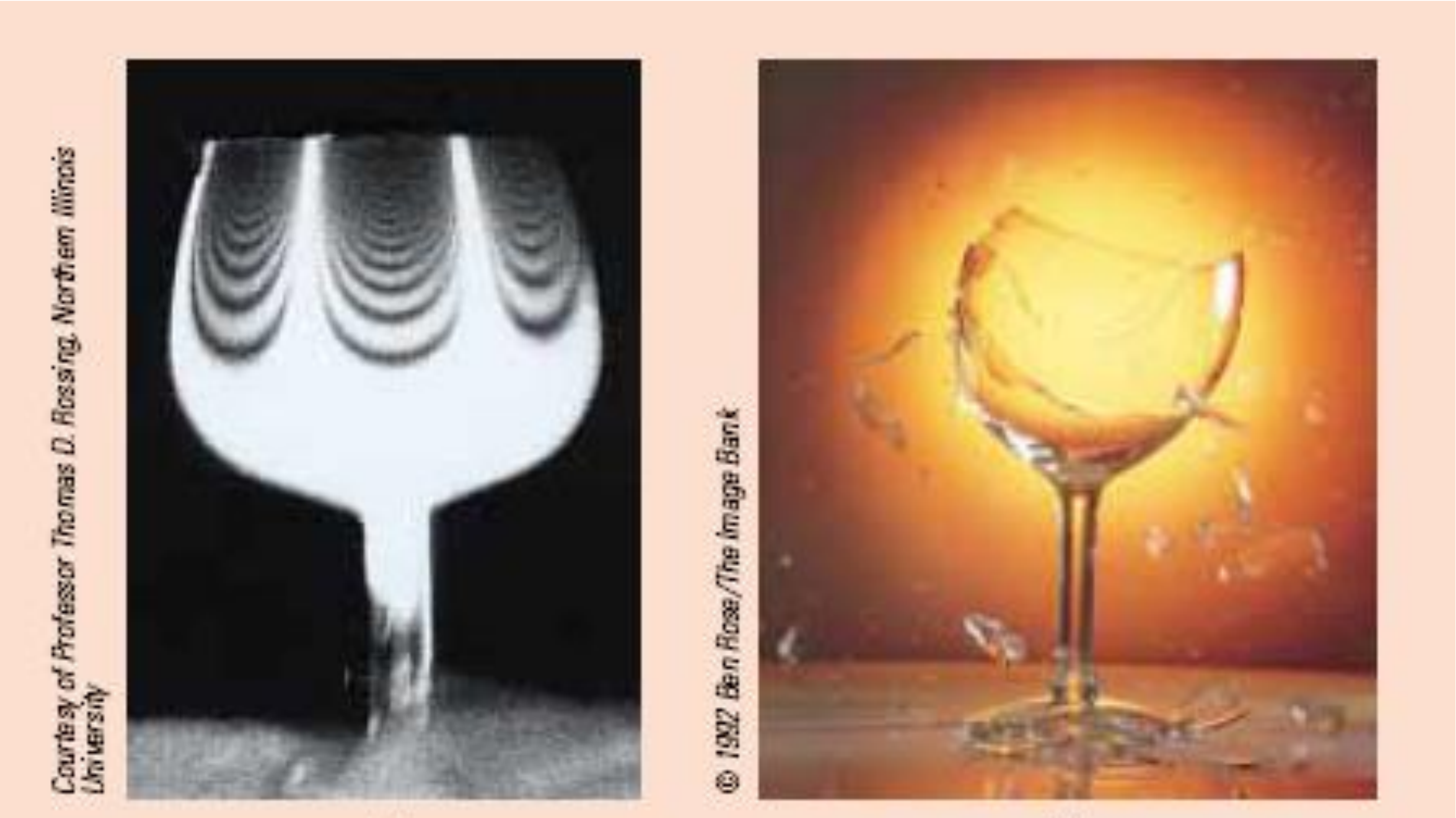
- If a periodic force is applied to such a system, the amplitude of the resulting motion is greatest when the frequency of the applied force is equal to one of the natural frequencies of the system
- This phenomenon, known as *resonance*
- These frequencies are often referred to as *resonance frequencies*

Resonance





Resonance



(a) (b)

Example

- The high E string on a guitar measures 64.0 cm in length and has a fundamental frequency of 330 Hz. By pressing down so that the string is in contact with the first fret (as shown in the figure), the string is shortened so that it plays an F note that has a frequency of 350 Hz. How far is the fret from the neck end of the string?

Example

For fundamental frequency, *n = 1*

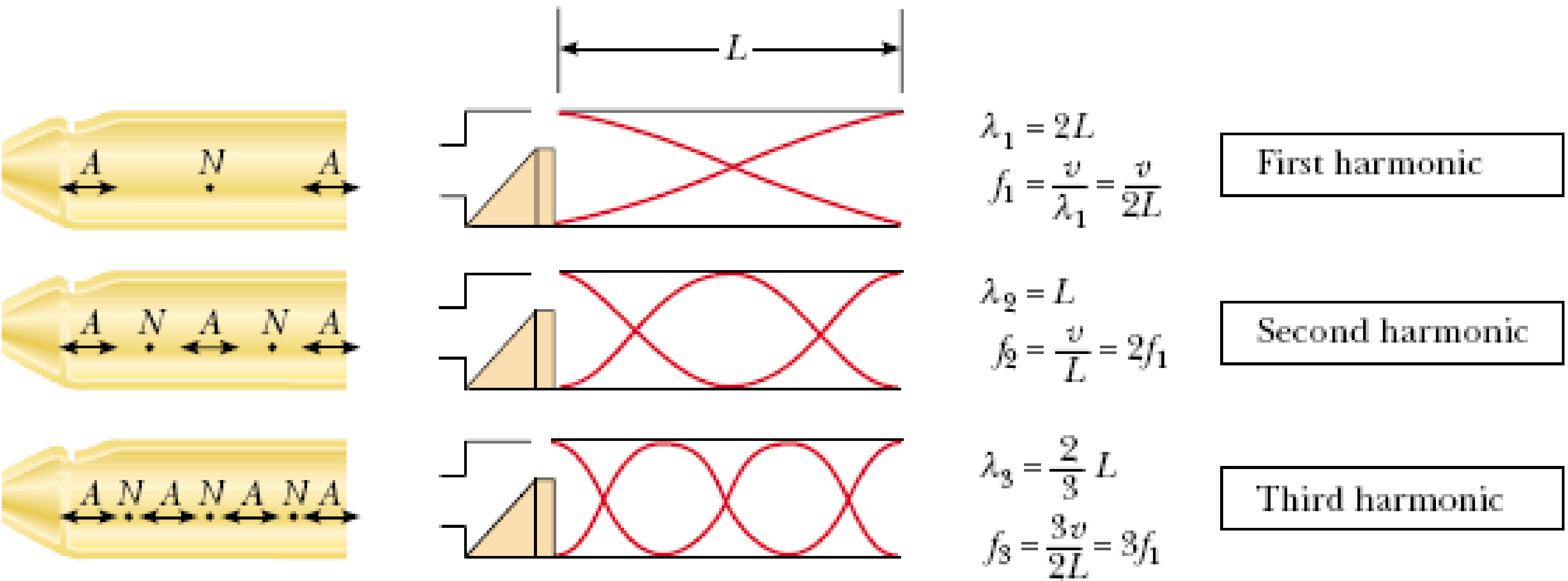
$$v = \frac{2L}{n} f_n = \frac{2(0.640\ \text{m})}{1} (330\ \text{Hz}) = 422\ \text{m/s}$$

Because we have not adjusted the tuning peg, the tension in the string, and hence the wave speed, remain constant

$$L = n\frac{v}{2f_n} = (1)\frac{422\ \text{m/s}}{2(350\ \text{Hz})} = 0.603\ \text{m} = 60.3\ \text{cm}$$

distance from the fret to the neck end of the string $3.7\ \text{cm}$

Standing Waves in Air Columns

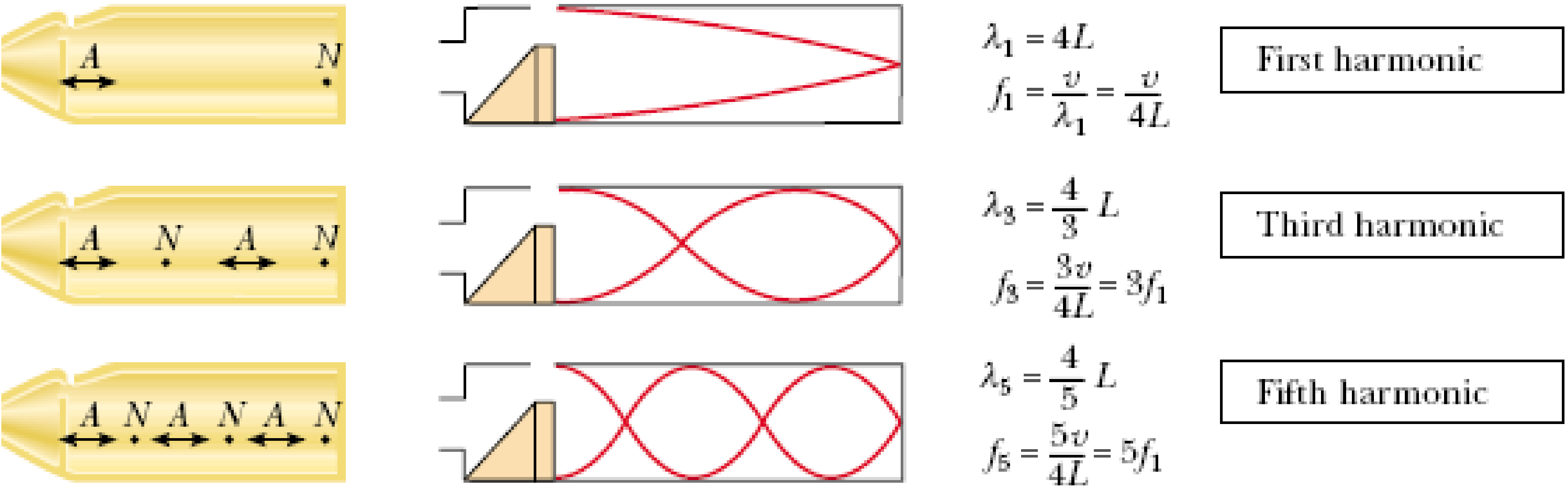


Standing Waves in Air Columns

$$f_n = n\frac{v}{2L} \qquad n = 1, 2, 3, \ldots$$

- In a pipe open at both ends, the natural frequencies of oscillation form a harmonic series that includes all integral multiples of the fundamental frequency

Standing Waves in Air Columns

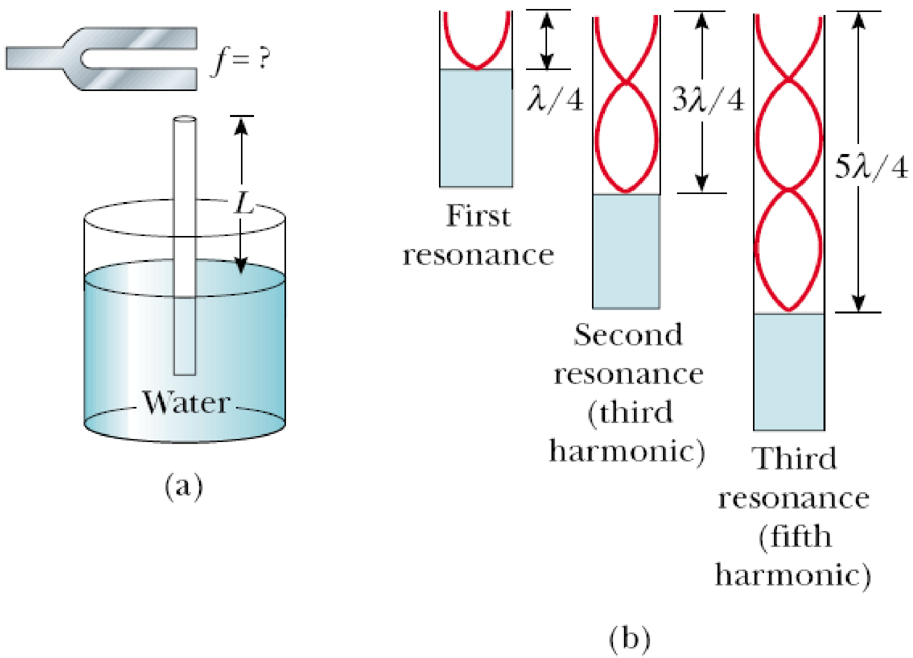


Standing Waves in Air Columns

$$f_n = n\frac{v}{4L} \qquad n = 1, 3, 5, \ldots$$

- In a pipe closed at one end, the natural frequencies of oscillation form a harmonic series that includes only odd integral multiples of the fundamental frequency

Example



- A simple apparatus for demonstrating resonance in an air column is depicted in the figure. A vertical pipe open at both ends is partially submerged in water, and a tuning fork vibrating at an unknown frequency is placed near the top of the pipe. The length *L* of the air column can be adjusted by moving the pipe vertically. The sound waves generated by the fork are reinforced when *L* corresponds to one of the resonance frequencies of the pipe.
  - For a certain pipe, the smallest value of *L* for which a peak occurs in the sound intensity is 9.00 cm. What are
    - **(A)** the frequency of the tuning fork
    - **(B)** the values of *L* for the next two resonance frequencies?

Example

(A) $$f_1 = \frac{v}{4L} = \frac{343\ \text{m/s}}{4(0.090\,0\ \text{m})} = 953\ \text{Hz}$$

(B) $$\lambda = 4L = 4(0.090\,0\ \text{m}) = 0.360\ \text{m}$$

$$L = 3\lambda/4 = 0.270\ \text{m} \quad \text{and} \quad L = 5\lambda/4 = 0.450\ \text{m}$$

Beats: Interference in Time

- interference phenomena
  - *spatial interference.*
    - Standing waves in strings and pipes
  - *interference in time* or *temporal interference*
    - results from the superposition of two waves having slightly *different* frequencies
      - beating
- Beating
  - the periodic variation in amplitude at a given point due to the superposition of two waves having slightly different frequencies

Beats: Interference in Time

$$y_1 = A\cos\omega_1 t = A\cos 2\pi f_1 t$$
$$y_2 = A\cos\omega_2 t = A\cos 2\pi f_2 t$$

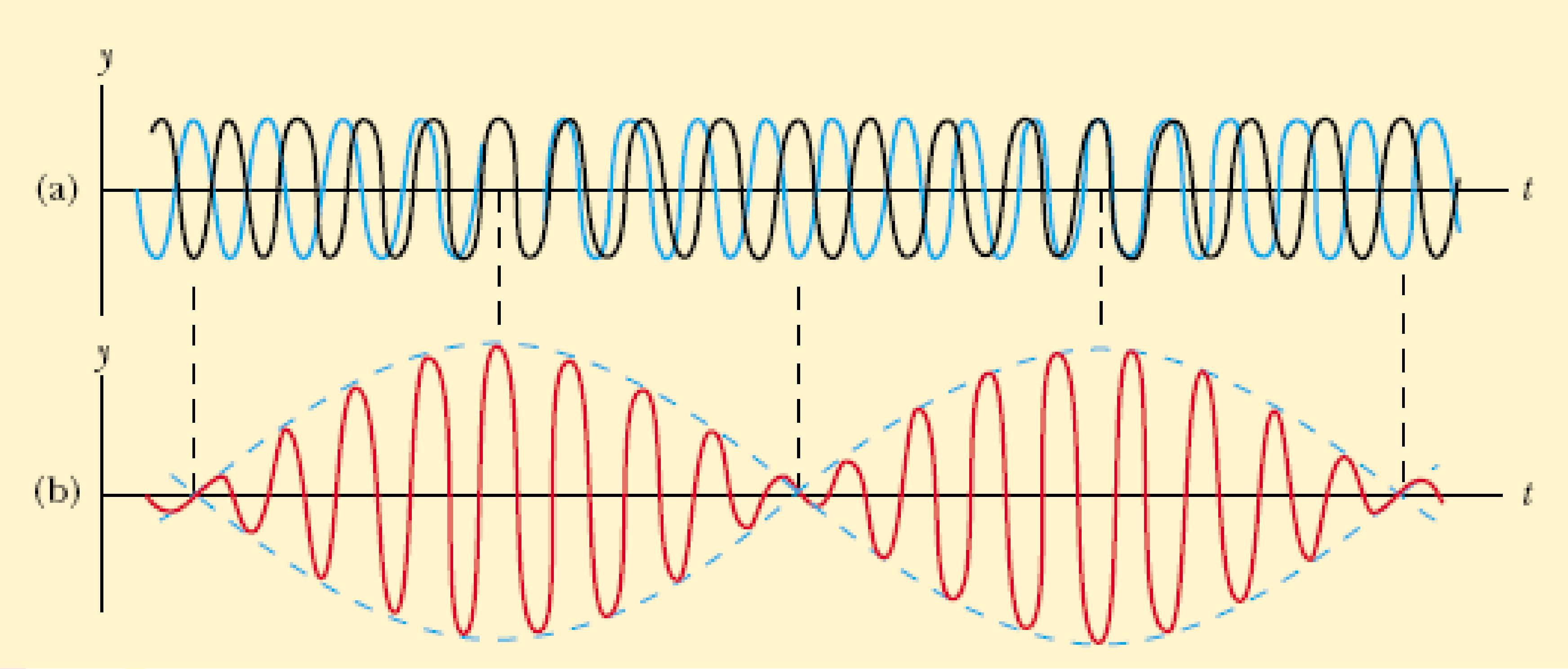
$$y = y_1 + y_2 = A(\cos 2\pi f_1 t + \cos 2\pi f_2 t)$$

$$\cos a + \cos b = 2\cos\left(\frac{a-b}{2}\right)\cos\left(\frac{a+b}{2}\right)$$

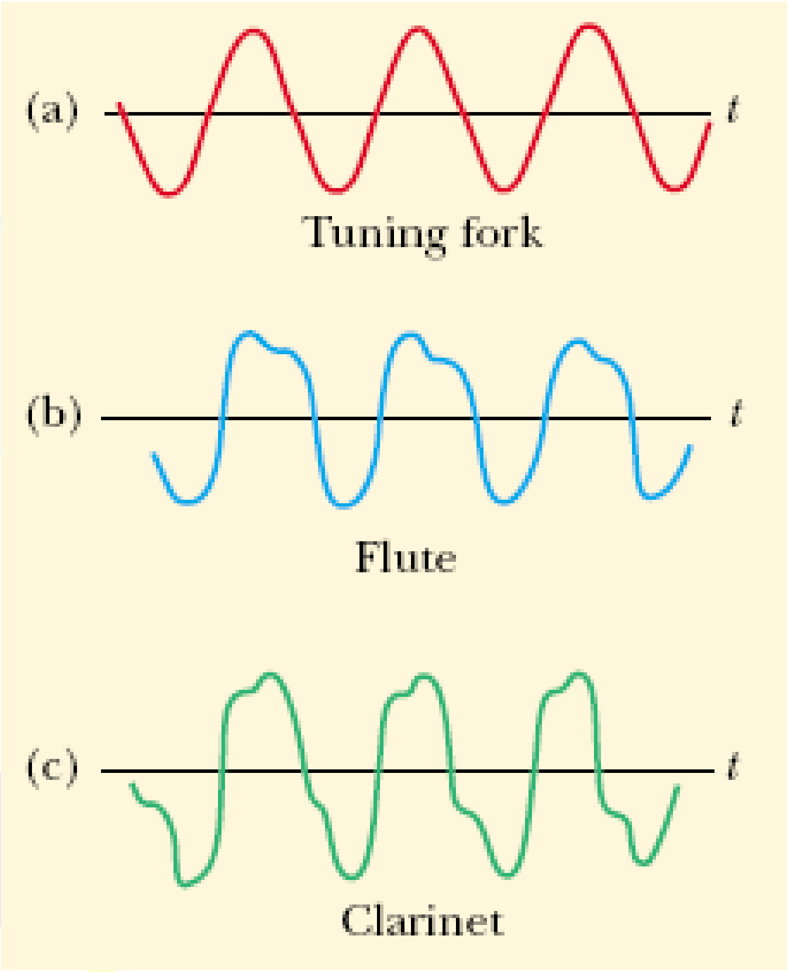
$$y = \left[2A\cos 2\pi\left(\frac{f_1 - f_2}{2}\right)t\right]\cos 2\pi\left(\frac{f_1 + f_2}{2}\right)t$$

$$A_{\mathrm{resultant}} = 2A\cos 2\pi\left(\frac{f_1 - f_2}{2}\right)t \quad \Rightarrow \quad f_{\mathrm{beat}} = |f_1 - f_2|$$

Beats: Interference in Time

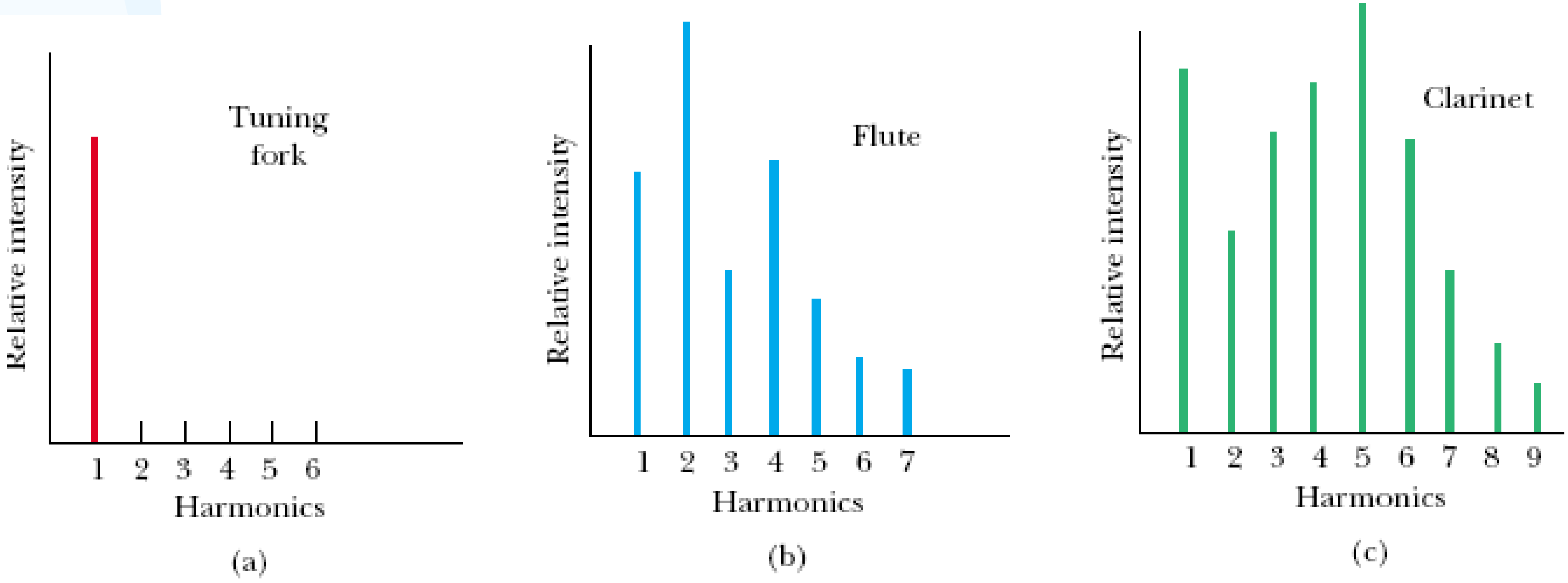


Nonsinusoidal Wave Patterns



- The wave patterns produced by a musical instrument are the result of the superposition of various harmonics
  - a *musical* sound
- A listener can assign a pitch to the sound, based on the fundamental frequency
- Combinations of frequencies that
- are not integer multiples of a fundamental result in a *noise*, rather than a musical sound
- The human perceptive response associated with various mixtures of harmonics is the *quality* or *timbre* of the sound

Nonsinusoidal Wave Patterns

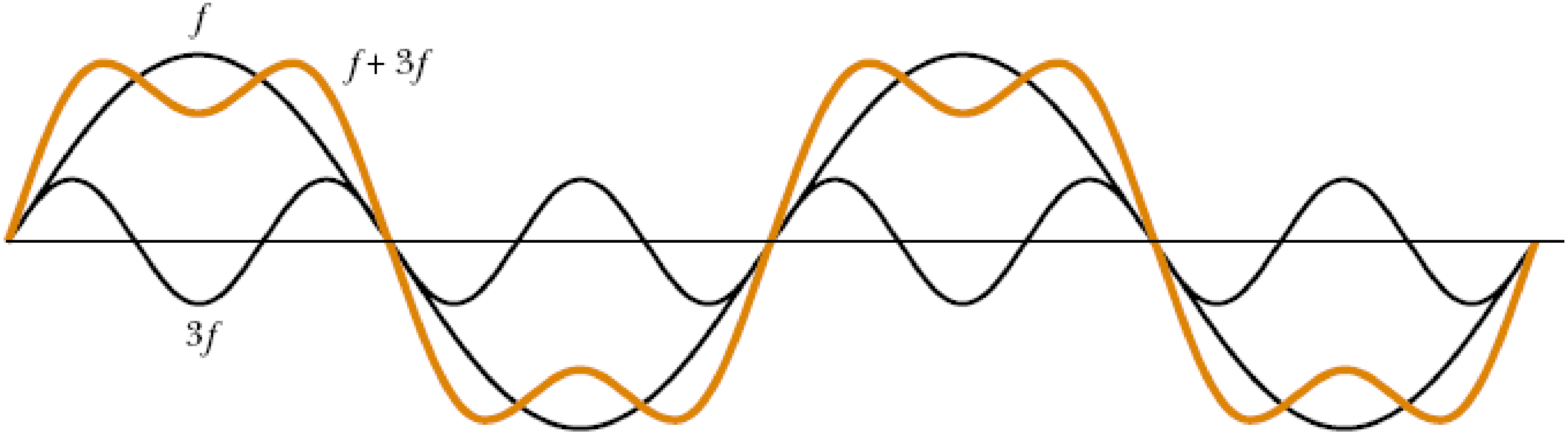


Nonsinusoidal Wave Patterns

- The corresponding sum of terms that represents the periodic wave pattern
  - called a *Fourier series*

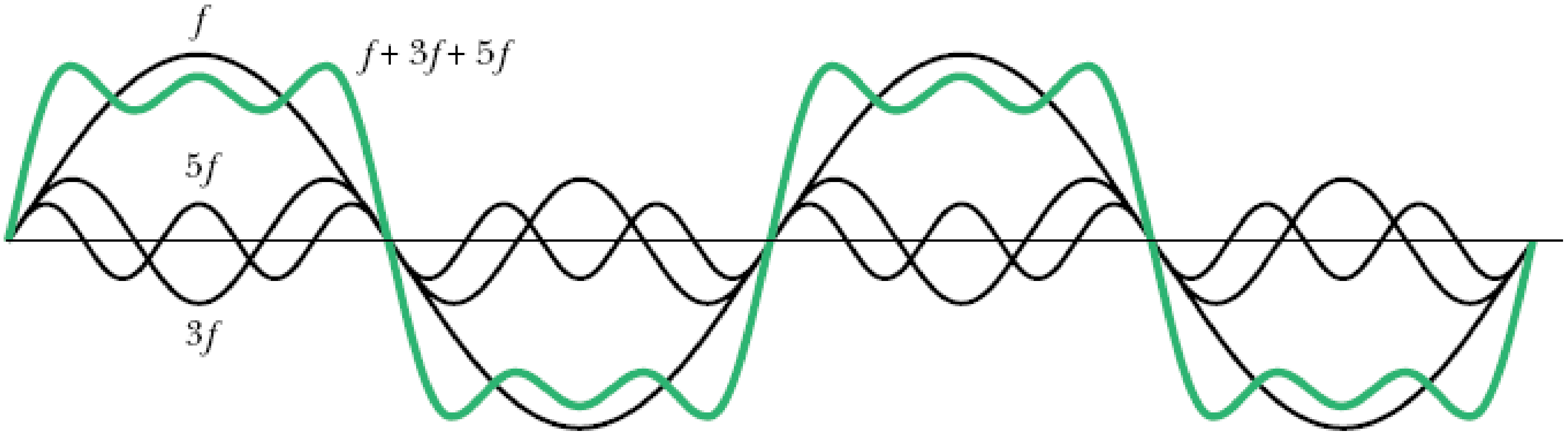
$$y(t) = \sum_n (A_n \sin 2\pi f_n t + B_n \cos 2\pi f_n t)$$

Nonsinusoidal Wave Patterns



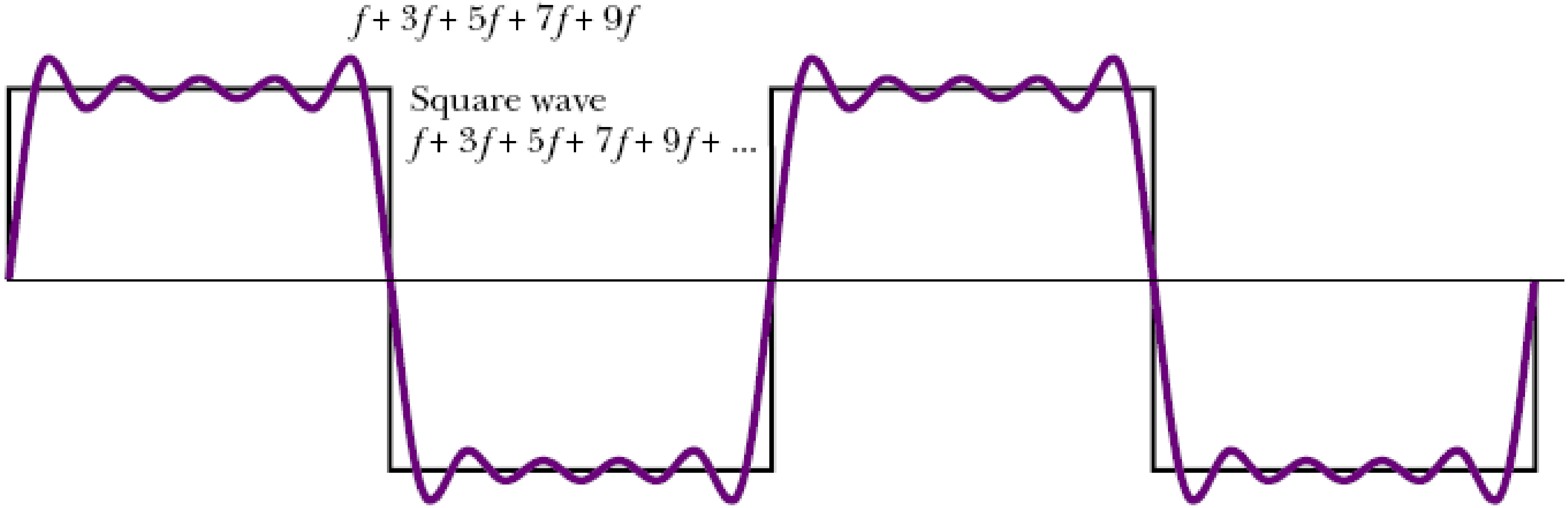
(a)

Nonsinusoidal Wave Patterns



(b)

Nonsinusoidal Wave Patterns



(c)